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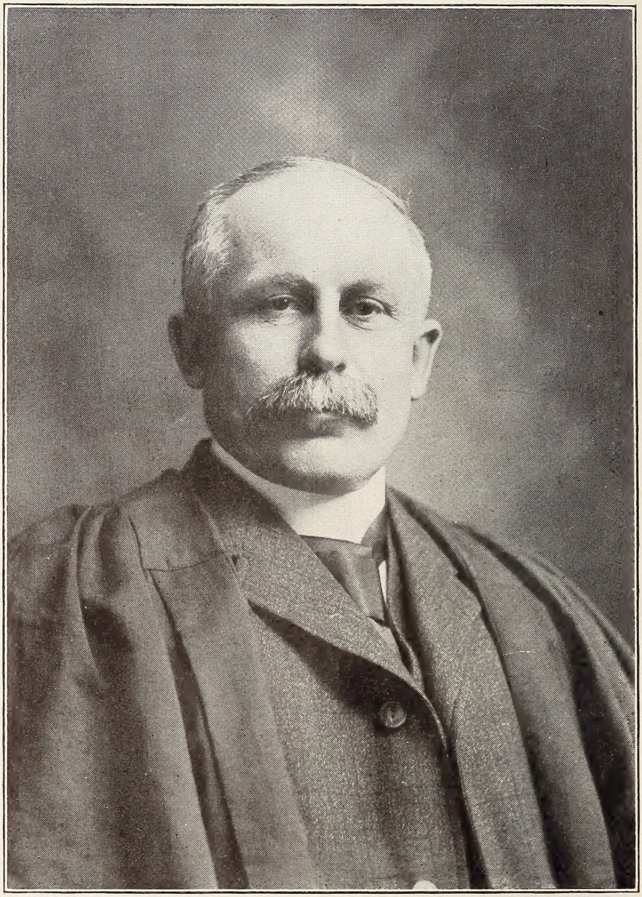

C. H. C. WRIGHT, B.A.Sc.

Supplement March Applied Science

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TORONTO ENGINEERING SOCIETY

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BRIEF SYNOPSIS OF THE COURSE OF LECTURES IN ANALYTICAL AND GRAPHICAL STATICS

As Arranged for the First Year.

C. H. C. WRIGHT, B.A.Sc.

Sub-Divisions of Mechanics.—The subject of Mechanics may be sub-divided into Kinematics, Statics and Dynamics. Kinematics treats of motion only; Statics, of force only; and Dynamics, of both force and motion.

Force—A force is that which moves or tends to move, or which changes or tends to change the motion of any body; e.g., the attraction of the earth or the expansion of steam, etc.

The elements which specify a force are (1) Magnitude (as 10 pds.); (2) Direction (as vertical); (3) Sense (as upward); and (4) Point or line of application.

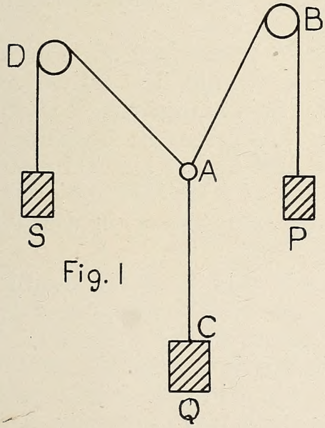

Fig. 1

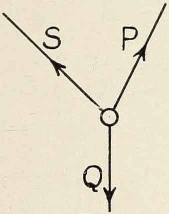

Fig. 2

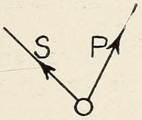

Fig. 3

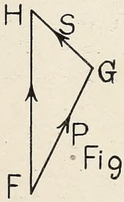

Fig. 4

Vector Quantity.—A Vector is any quantity which has magnitude, direction and sense; e.g., force, displacement, acceleration, etc.

As a finite straight line with an arrow-head has magnitude, direction and sense, it may represent any vector quantity.

Vector Polygon.—If from any initial point, a set of lines be drawn to represent a series of vector quantities in such a manner that their senses are continuous, they will form a polygon known as a vector polygon.

The line joining the initial to the final point (with the sense from the initial towards the final point) will represent the resultant. When the vector quantities are forces, the closing line of the vector polygon represents the resultant force; i.e., the single force capable of producing the same effect as the set of forces. The proof of the above is experimental.

Experimental Proof.—Fasten three strings, *AB*, *AC* and *AD* together at the point *A*, as indicated in Fig. 1.

Pass the string *AB* over a frictionless pulley and attach the end *B* to a known weight *P*. Similarly, pass *AD* over a second pulley and fasten the end *D* to a known weight *S*. Fasten the end *C* of the string *AC* to a known weight *Q*.

Now, let the system adjust itself and the point *A* come to rest. The knot *A* is acted on by three forces (caused by the tensions of the three strings) of magnitude *P*, *Q* and *S*, directions parallel to the three strings, and senses as indicated in the accompanying diagram (Fig. 2).

Now discuss the two forces *P* and *S* as in Fig. 3.

Draw the vector polygon *FGH* where *FG* represents the force *P* and *GH* — *S*. The line *FH* must then represent the resultant.

If the experiment indicated were performed and the vector polygon drawn to scale, the line *FH* would be vertical and its length represent the magnitude *Q*. This being true, it is obvious that a single force represented by *FH* might be applied to the knot *A* instead of the two forces *P* and *S*, and leave the point in a condition of rest; i.e., the resultant of *P* and *S*, as determined by the vector polygon, produces the same effect on the point *A* as the original forces.

Equilibrium.—A set of forces is said to be in equilibrium when they do not move or tend to move, nor change nor tend to change, the motion of the body upon which they act. Their resultant is, therefore, 0, and the final point of the vector polygon for such a set of forces must coincide with the initial point.

A weight of 12 lbs. is fastened to a string attached to a peg. A second string is tied to the first between the peg and the weight, and pulled horizontally until the part of the first string above the second makes an angle of 60° to the horizontal. Determine the tensions of the strings.

The annexed diagram (Fig. 5) represents the above condition. Consider the forces acting on the knot. There are three

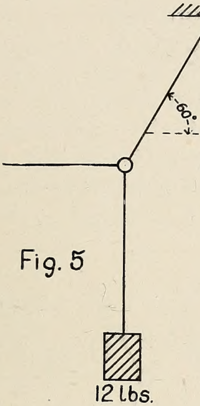

Fig. 5

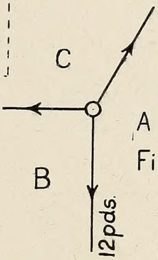

Fig. 6

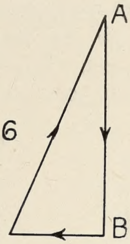

Fig. 7

as illustrated in Fig. 6: (1) The pull *AB* of the vertical string; (2) the pull *BC* of the horizontal string; and (3) the pull *CA* of the inclined string. Draw the vector polygon *ABC* (Fig. 7) for these three forces. As they are in equilibrium, the vector polygon closes and *BC* and *CA* of the vector polygon represent the forces *BC* and *CA*. To obtain the magnitudes of these forces, measure the lengths of the sides of the vector polygon. Because the angles *B* and *C* are 90° and 60° respectively, and *AB* represents a force of 12 pds., therefore *BC* represents a force of $4\sqrt{3}$ pds., and *CA* of $8\sqrt{3}$ pds.

Cantilever.—Let the accompanying diagram (Fig. 8) represent a cantilever supporting a load of *W* lbs. It is required to find the stresses in the various members of this truss.

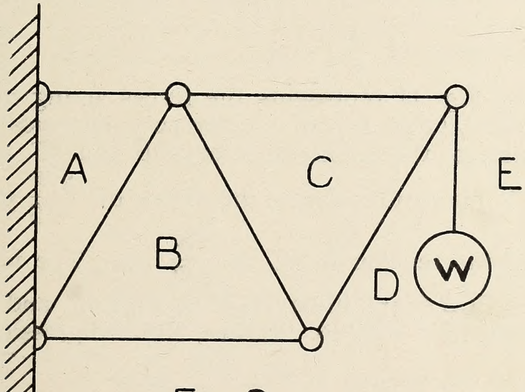

Fig. 8

Consider first, the forces acting on the point *CDE* (Fig. 9). There are three forces: (1) The vertical pull of the member *DE* (*W* pds.); (2) a force caused by the member *CD*; (3) a force caused by the member *CE*. The magnitudes and senses of the latter two forces are unknown; but, their directions are those of the members *CD* and *CE* respectively. From the point *D* (Fig. 10) draw *ED* to represent the known force *ED*; and through *E* and *D*, draw *EC* and *DC* parallel to the direction of the two unknown forces. These lines will intersect at the point *C*, and the vector polygon for the three forces then reads *ED*, *DC*, *CE*, and the lines *DC* and *CE* represent the forces *DC* and *CE* completely. Hence the member *DC* pushes on the point and is in compression to the extent of $\frac{2W}{\sqrt{3}}$ pds., and the member *CE* in tension to the extent of $\frac{W}{\sqrt{3}}$ pds.

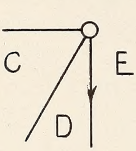

Fig. 9

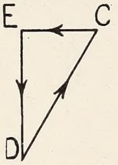

Fig. 10

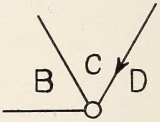

Fig. 11

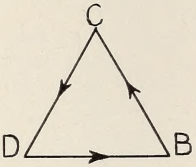

Fig. 12

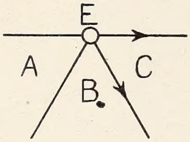

Fig. 13

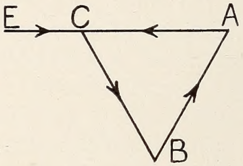

Fig. 14

Similarly, Fig. 11 represents the forces acting on the point *BCD*, and *CDBC*, (Fig. 12), the vector polygon gives the magnitude and senses of the unknown forces *DB* and *BC*. Hence, the member *DB* is in compression to the extent of $\frac{2W}{\sqrt{3}}$ pds. and the magnitude of the tension in *BC* is $\frac{2W}{\sqrt{3}}$ pds. The forces acting on the point *ECBA* are illustrated in Fig. 13 and the vector polygon for them in Fig. 14. Hence, the compression in *AB* is $\frac{2W}{\sqrt{3}}$ pds., and the tension in *AE*, $\frac{3W}{\sqrt{3}}$ pds.

Resolved Parts.—If two forces *P* and *Q* have a resultant *R* they are called the components of *R*. When their directions are at right angles to each other they are called the resolved parts of *R*.

The algebraic sum of the resolved parts of any set of forces in any direction is equal to the resolved part of their resultant in the same direction.

$$\Sigma X = X_R \quad \Sigma Y = Y_R.$$

Moment.—The moment of a force about a point is the product of the magnitude of the force, and the perpendicular distance of the point from the force, and is positive or negative as the force tends to turn the body on which it acts about the selected point, with or contrary to the hands of a watch.

Let P and Q (Fig. 15) represent two known forces, and A any point. It is required to prove that the algebraic sum of the moments of P and Q about A is equal to the moment of their resultant about A.

$M_P + M_Q = M_R$

$\Sigma M = M_R$

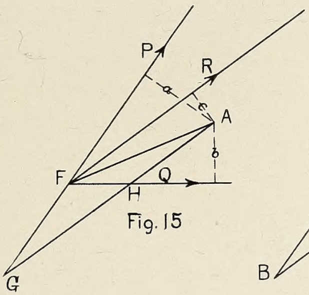

Fig. 15

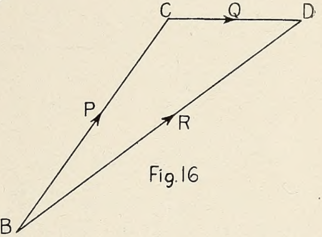

Fig. 16

Draw the vector polygon BCD (Fig. 16). The resultant will be represented by BD, and will act through F, the intersection of P and Q. Through F draw a line parallel to BD. Through A draw a line parallel to the resultant, intersecting the directions of P and Q at G and H. The Δ *le* GFH may then be considered as a vector polygon for the three forces P, Q and R. From A, drop perpendiculars a, b and c on P, Q and R. Join AF.

$$M_P = +P \times a = +GF \times a = +2 \times \text{ area of } \Delta\, le\ GFA;$$

$$M_Q = -Q \times b = -FH \times b = -2 \times \text{ area of } \Delta\, le\ FHA;$$

$$M_P + M_Q = +2\ le\ GFA - 2\ \Delta\, le\ FHA = +2\ \Delta\, le\ GHF$$

$$= HG \times c = +R \times c = M_R$$

Hence, $M_P + M_Q = M_R$.

As this is true for two forces, it is true for any number of forces, i.e., the algebraic sum of the moments of a set of forces about any point is equal to the moment of their resultant about the same point, or

$\Sigma M = M_R$.

Method of Sections.—Consider the truss illustrated in Fig. 17 as a rigid body, and take moments about the point NDF. The forces acting on this rigid body are DF, FG, GH, HA, AB, BC

and CD. Because they are in equilibrium $\Sigma\ M = 0$, therefore

$$DF \times 0 - 1200 \times 10 - 1200 \times 20 + HA \times 30$$
$$- 600 \times 25 - 600 \times 15 - 600 \times 5 = 0$$

or $HA \times 30 = +63000$

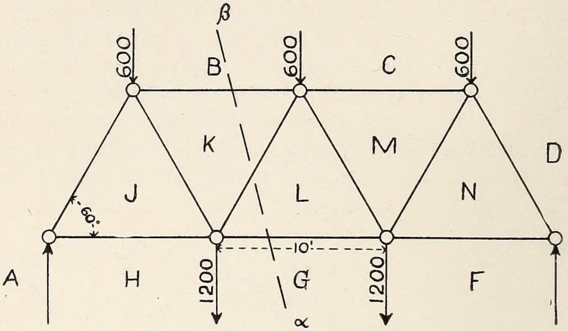

Fig. 17

Hence, $HA = 2100$

Again, because $\Sigma\ Y = 0$,
therefore, $+HA - 600 - 1200 - 600 - 1200 - 600 + DF = 0$
or $DF = 2100$.

Let the plane $\alpha\beta$ cut the truss as indicated, the consider the portion of the truss to the left as a rigid body. The forces

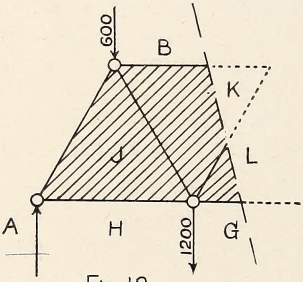

Fig 18

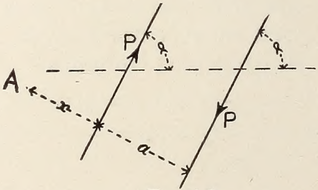

Fig. 19

acting on this rigid body are GH, HA, AB, BK, KL and LG, as indicated in Fig. 18. Because these forces are in equilibrium $\Sigma\ Y = 0$
therefore $-1200 + 2100 - 600 + 0 + Y_{KL} + 0 = 0$
therefore, $Y_{KL} = -300$ pds.
that is, $KL \sin. 60° = -300$
or $KL = -200\sqrt{3}$ pds.

This $-ve$ sign means that the portion of the member KL to the right of $\alpha\beta$ is pushing on the part to the left, and the member KL is, therefore, in compression, the magnitude of the compression being $200\sqrt{3}$ pds.

Again, $\Sigma M = 0$

Take moments about the point $GHJKL$

$$1200 \times 0 + 2100 \times 10 - 600 \times 5 + BK \times 5\sqrt{3} + KL \times 0 + LG \times 0 = 0$$

$BK = -1200\sqrt{3}$

This $-ve$ sign means that the moment of BK is $-ve$, and BK, therefore, has a sense towards the left. The member BK is, therefore, in compression to the extent of $1200\sqrt{3}$ pds.

Again $\Sigma X = 0$

therefore, $0 + 0 + 0 + X_{BK} + X_{KL} + X_{LG} = 0$

$-1200\sqrt{3} - KL \cos. 60^\circ + LG = 0$

therefore, $LG = 1200\sqrt{3} + 100\sqrt{3} = 1300\sqrt{3}$

hence, the member LG is in tension to the extent of $1300\sqrt{3}$ pds

Fink Roof Truss.—For the solution of the Fink Roof Truss, see Transactions of the Engineering Society, No. 17.

Wind Pressure.—$v = \sqrt{2gh}$

$$p = h \times \text{(the wt. of a unit of vol.)}$$

therefore $$v^2 = \frac{2gp}{\text{(wt. of unit of vol.)}}$$

or $$p = \frac{v^2}{2g}\text{(wt. of unit of vol.)}$$

In the above, it is customary to use the foot and the second as the units of measurement; but the velocity of the wind is usually expressed in miles per hour.

When V represents the velocity of the wind in miles per hour

$$v = \frac{V \times 5280}{60 \times 60}$$

therefore $$p = \frac{V^2 \times 5280^2 \times \text{wt. of a cub ft. sf air}}{60^4 \times 2g}$$

One cubic foot of air at 60° and 760 mm. weighs 0.078 lbs. For a maximum value, take 0.1 lbs. as the weight of a cubic foot of air, and the equation becomes

$$p = \frac{V^2 \times 5280^2 \times 0.1}{60^4 \times 64} = .0033\,V^2.$$

Hence, the pressure in pds. per square foot on a surface at right angles to the direction of the wind, is given by the equation $p = .0033V^2$ where V is in miles per hour.

The observed values of Prof. Marvin show $p = .004V^2$, while those of Mr. Langley give $p = .00315V^2$.

If the pressure on a surface inclined to the direction of the wind be proportional to the number of particles of air impinged on its surface, then $p' = p \sin. \alpha$ where p' is the pressure perpendicular to the plane, in pds. per sq. ft., when the plane is inclined to the horizontal at an angle α, the wind being horizontal. Therefore,

$$p' = .0033\ V^2 \sin \alpha.$$

Couple. $\Sigma X = 0$; $\Sigma Y = 0$; $\Sigma M = C$.—When two forces of equal magnitude have parallel directions and opposite senses, as indicated in Fig. 19, they are called a couple.

$$\Sigma X = +P \cos.\ \alpha - P \cos.\ \alpha = 0$$
$$\Sigma Y = +P \sin.\ \alpha - P \sin \alpha = 0$$

Take moments about any point A,

$$\Sigma M = -Px + P(x + a) = +Pa.$$

As both P and a are constant, the moment of a couple about any point is constant and is equal to the product of one of the forces and the perpendicular distance between them.

Graphical Determination of Resultant Force.—Let P, Q and S represent three forces acting on a rigid body in one plane, as indicated in Fig. 20. Draw the vector polygon $ABCD$, as in Fig. 21; then AD will represent their resultant in magnitude, direction and sense. Select any point O and join O with A, B, C and D. Select any point F in the line of direction of the force P, and at F, imagine P to be replaced by a pair of components represented by AO and OB. Produce the direction of OB to intersect that of Q at G, and at G imagine Q to be replaced by its components BO and OC. Produce the direction of OC to intersect S at H, and at H imagine S replaced by its components CO and OD. Now, the original forces P, Q and S have substituted for them their equivalents in AO, OB, BO, OC, CO and OD. Two of these six forces, OB and BO act with equal magnitude and opposite senses in the same direction, and are, therefore, in equilibrium. Similarly, OC and CO are in equilibrium. Hence, AO acting at F and OD acting at H are equivalent to the original

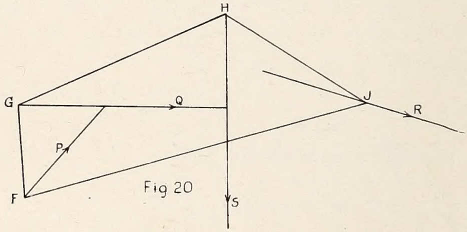

Fig 20

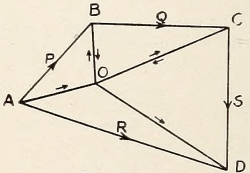

Fig. 21

forces. The resultant of *AO* and *OD* will act, through their intersection *J*. Hence, *R* the resultant of *P*, *Q* and *S* must act through *J*.

Funicular Polygon.—The figure *FGHJ* is called the funicular polygon.

General Conditions.

Analytical	Graphical
$\Sigma\ X = X_R$ $\Sigma\ Y = Y_R$	Vector polygon gives the Resultant
$\Sigma\ M = M_R$	Funicular Polygon gives the position of the Resultant

Conditions of Equilibrium.

$\Sigma\ X = 0$ $\Sigma\ Y = 0$	Vector Polygon closes.
$\Sigma\ M = 0$	Funicular Polygon closes.

Couple.

$\Sigma\ X = 0$ $\Sigma\ Y = 0$	Vector Polygon closes.
$\Sigma\ M = C$	Funicular Polygon open.

The Simple Horizontal Beam.—Let the annexed diagram (Fig. 22) represent a simple, horizontal beam resting on two supports and carrying a load of *W* lbs., distance *k* from the left abutment *A*. Imagine a vertical plane $\alpha\beta$ distant *x* from *A*.

V. S. F.—The vertical shearing force (V. S. F.) at any plane is, by definition, the algebraic sum of the external forces acting on the beam to the left of the plane.

When $x < k$, the V. S. F. at $\alpha\beta = +A = \frac{l-k}{l}\,W$.

Similarly, when $x > k$, the V. S. F. $= +A - W = -\frac{k}{l}\,W$.

Hence, the ordinates to the lines *CD* and *FG* (Fig. 22) represent the values of the V. S. F.

B. M.—The bending moment (B.M.) at any plane $\alpha\beta$ is the algebraic sum of the moments of the external forces to the left of the plane about any point in the plane.

When $x = 0$, B.M. $= 0$

When $x < k$, B.M. $= +Ax$, and may be represented in Fig. 21 by the area of the rectangle *CH*.

When $x > k$, B.M. $= A \times x - W\ (\mathrm{x} - k) = A \times \mathrm{k} - W \times \frac{k}{l}(x - k)$ and may be represented, in Fig. 22, by the rectangle *CJ* — the area of the rectangle *FL*; i.e., the area of the V. S. F. diagram to the left of any plane $\alpha\beta$ represents the B. M.

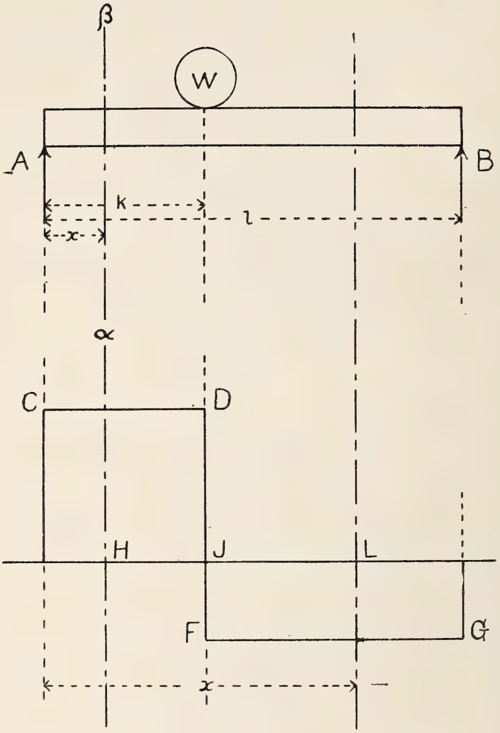

Fig. 22

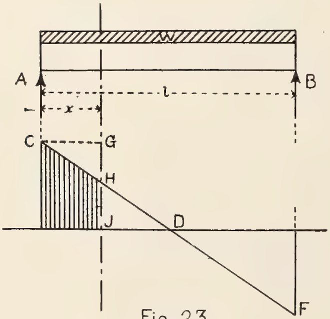

Fig. 23

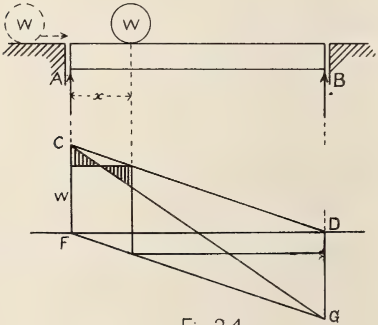

Fig. 24

Let Fig. 23 represent a horizontal beam supporting a uniformly distributed load of W lbs. The V. S. F. at any plane

$$\alpha\beta = +A - \frac{x}{l} W = + \frac{W}{2} - \frac{x}{l} W$$

When $x = 0$, V. S. F. $= + \frac{W}{2}$

When V. S. F. $= 0$, $x = \frac{l}{2}$

Hence, the ordinates to the line CDF will represent the Vertical Shearing Forces.

The B. M. at any plane $\alpha\beta = A . x - \frac{x}{l} . W . \frac{x}{2}$

$$= \frac{W}{2} x - \frac{W}{2l} x^2$$

The B. M. at $\alpha\beta$ may be represented by the area of the rectangle CJ less the area of the triangle CGH or by the area of V. S. F. diagram to the left of $\alpha\beta$.

Live Load.—As the load W moves across the beam from A to B (Fig. 24), let x represent the distance of the load from A.

The V. S. F. to the left of the load $= + A = \frac{l - x}{l} W$.

When $x = 0$, V. S. F. $= +W$.

When $x = l$, V. S. F. $= 0$.

When the load is at a distance x from the left abutment, the V. S. F. to the left of the load will be represented by the ordinate of the line CD directly under the load.

Similarly, the V. S. F. to the right of the load $= +A - W$

$$= - \frac{x}{l} .W$$

When $x = 0$, V. S. F. $= 0$.

When $x = l$, V. S. F. $= -W$.

Hence, the ordinates of the line FG will represent the V. S. F. to the right of the load.

Join CG and the B. M. at $\alpha\beta$ distant x from A, when W is at

$$\alpha\beta = +Ax = + \frac{l - x}{l} Wx.$$

Therefore, the area between the line CG and the axis of X to the left of $\alpha\beta$ represents the B. M. The maximum B. M. will occur at the centre of the beam when the load is over that point.

The Ordinates of the Funicular Polygon represent the Bending Moments.—Let the annexed diagram (Fig. 25) represent a horizontal beam supporting four loads. Draw the vector polygon ABC, etc. Next, draw the funicular polygon, FGH, etc. As the forces are in equilibrium, the funicular polygon closes. Hence, the direction of the line FP and the point L in the vector polygon. Now, draw the V. S. F. diagram, the areas in which will also represent the B. M.

Let $\alpha\beta$ be any plane between the loads AB and BC. Let it intersect the funicular polygon at the points J and K. The forces acting on the portion of the beam to the left of $\alpha\beta$ are LA and AB. The algebraic sum of their moments about any point in $\alpha\beta$ is equal to the moment of their resultant about the same point; but, the resultant of these forces is represented by LB in the vector polygon and acts through the point M as given by the funicular polygon. Now, the Δ le JMK is similar to the Δ le LOB. Hence, $KJ \times h' = LB \times h$; but, $LB \times h =$ the B. M. Therefore, KJ, the ordinate of the funicular polygon may represent the B. M. and the scale of B. M.'s will be the scale of forces $\times$ the scale of length $\times$ h'.

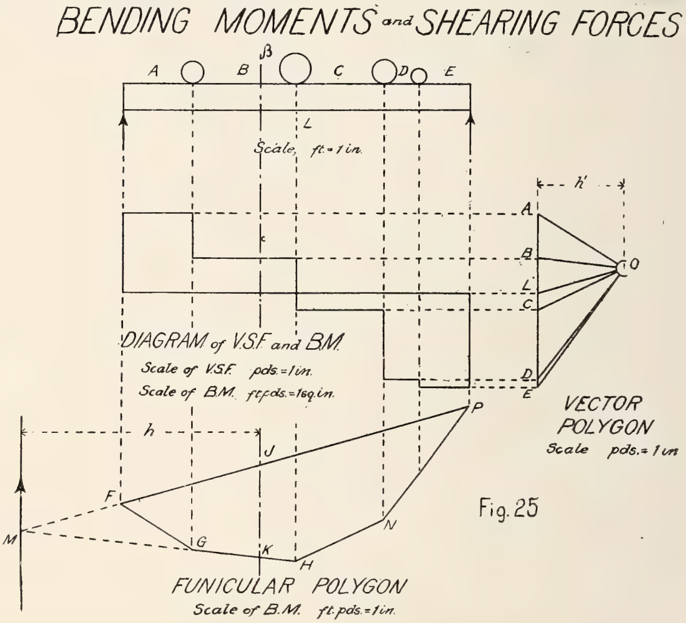
BENDING MOMENTS and SHEARING FORCES
β
A
B
C
D
E
L
Scale, ft. = 1 in.
h'
A
B
L
C
D
E
O
DIAGRAM of V.S.F. and B.M.
Scale of V.S.F. pds. = 1 in.
Scale of B.M. ft.pds. = 1 sq. in.
h
J
P
F
M
G
K
H
N
VECTOR POLYGON
Scale pds. = 1 in
FUNICULAR POLYGON
Scale of B.M. ft.pds. = 1 in.

Fig. 25

In addition to the work indicated by the above synopsis, the values of the V. S. F.'s and B. M.'s are determined for a series of moving loads (the engine diagram), and the following are discussed at some length: friction, the inclined plane, the pulley and the windlass.

The stresses in the members of the following trusses are also determined: Cantilever, King Post, Queen Post, Howe, Pratt, Warren, German Roof Truss, French Roof Truss, and Fink Roof Truss.

*MATHEMATICS OF TREE GROWTH

B. E. FERNOW

Dean, Faculty of Forestry

It is of course generally known that trees grow in height by adding story upon story of annual shoots which arise from buds, and in girth or diameter by adding layer upon layer of wood, cornucopia-like, upon the wood of the preceding years; the cambium layer of cells, lying between the bark and the wood, giving off wood cells and bark cells, and thus increasing the girth. This layer of cambium cells represents the principal living portion of the tree, for the wood, the deposit of annual layers, which on a cross-section appears as the familiar annual rings, is mostly dead material, an accumulation of cells and tissues that have lost their protoplasmic contents and their ability to divide and grow.

The botanist studies *how* trees grow, but the forester goes a step farther and inquires *how much* trees grow; the quantity or amount of their performance—their increment, their "yield," is his main interest. Through laborious measurements and classification of his results he has succeeded in recognizing and formulating the laws of growth in quantitative direction.

The fact not only that the increment in girth takes place by defined annual additions, but that by the difference in structure of the initial or spring wood, formed in the beginning of the season, and of the later-formed summer wood, each annual ring can be recognized as such—these facts permit us to relate increment to time, and to study the quantitative performance from year to year, or from period to period on a cross-section as functions of time. By stem analysis, as the forester calls the operation, the whole history of growth of the tree in height, diameter, form (i.e. diameter at different heights) and volume, is revealed; and from its behavior, or rather from the measured average performance of many trees grown under similar conditions, the behavior of others of the same species under the same conditions may be predicted. The methods of measuring have developed into a technical discipline, "forest mensuration," which forms one of the important equipments of the forester. Here I may only hint at the method of unravelling this history of tree growth. It is done by cutting full-grown trees into sections of three to six or more feet in length, counting and measuring on each cross-section the width of annual rings belonging to each period of its age, say from ten to ten years, and tabulating the changes in height, diameter, form and volume in tables or curves.

To be sure, not only do different species grow at different rates due to their inherent disposition, but each species in itself varies its rate of growth according to climate, soil and light conditions. Hence a classification of the measurements according to "site" classes must be made for each species, and the so-called "yield tables," which record the measurements, are constructed with due regard to the differences of site.

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*Read before the Canadian Institute, January, 1908.

A site in the forester's parlance, then, is the combination of climatic and soil conditions which produce a certain growth energy. In nature there is of course an endless variety of sites, but within a given climatic region a differentiation into five site classes has been usually found sufficient for practical purposes.

Light, next to soil and climate, is a most potent stimulus to assimilation of food materials by the foliage, and hence influences growth. For this assimilation we find that some species require much light, are intolerant of shade, others are satisfied with small amounts, or rather can utilize rays which the first cannot utilize; they can assimilate under the shade of others. There are various degrees of shade endurance possessed by the various species, according to which they can be grouped into light-needing and shade-enduring, and according to whether they have more or less light they grow more or less.

Trees behave very differently as regards growth when growing in the open than when grown in the forest, due largely to difference in light conditions. And, since the forester is interested mainly in the trees as grown in the forest, his measurements are made under forest conditions, and refer to trees in the forest. His deductions may therefore not be applied directly to single lawn trees which are a law unto themselves.

In discussing the laws of tree growth the forester has always in mind a close stand of trees, where crown touches crown, and the whole ground is covered by tree growth. In formulating the laws he must also predicate a pure stand of one species since the mixed stand of many species introduces too many variables.

He constructs, therefore, his yield tables for single species in closed stand, i.e., perfect condition, as a normal standard with which he can compare actual conditions. In such yield tables will be recorded from decade to decade the progress in height, in diameter or in basal area, and the increment in volume on various sites—not only of the average tree but of a whole acre of trees of the same kind.

The object of such yield tables is to enable the forester to relate production to time, and to predict the amount of the harvest if cut at various times, and to choose the most profitable time for such cutting. For maturity of the forest crop is not a natural condition, but an economic condition, which is determined by calculation, either on the basis of maximum volume growth or maximum value increment, or of still more complex *financial* considerations.

The measurements, to be sure, can only be approximations, more or less close; and as seasons vary, and with them the increment, in order to get a clear picture of the lawful progress, it is preferable not to state yearly but only periodic performance, the period chosen being a decade or at best a quinquennium. The annual or *current* increment would then appear really as an average periodic one, the average of ten years; and the *total average* increment would be the amount found at any given period, divided by the number of years up to that period. By plotting the actual findings of many measurements, in a system of co-ordinates representing time and amounts, it is possible to strike an average which in a curve or straight line represents graphically the changes, and from the form of the curve we can get an idea of, and indeed calculate mathematically the rate of progress. A concave trend of the curve, as is well known, represents an increasing, a straight line a steady rate, and a convex form a decreasing rate of progress.

Just as man and other animals exhibit periods or stages of development so we can discern periods of growth in the tree, the infantile and juvenile, the adolescent, the virile and the senile stage, each varying in the rate of growth and character of development.

We note that in the so-called great period of development in tree growth, from the seed to death, at least three or four stages can be recognized during which the rate of growth in all directions varies. From the very slow seedling stage some species pass through a short, others through a longer brush-wood stage, when the tree grows into branches rather than bole. Then usually follows a rapidly increasing rate through the pole-wood stage, when a definite bole is formed, the lower branches are lost, and a definite crown develops, to be succeeded by a steady progress of increment in the young timber stage; finally in old timber the rate retards and nearly comes to cessation. The juvenile period, varying greatly in length for different species, different sites and in different directions of growth, is the one that is least amenable to law; just as among boys the duration of this period of development varies greatly and is very much dependent on outward conditions. But for this period, which withdraws itself from definite mathematical consideration, all species would appear to behave very much alike.

While it is a law of nature that nothing can stand still, but must either grow or die, in one direction, height growth, there is, as we all know, an absolute limit—trees do not grow into the sky; but in diameter and volume there is no cessation of increment until the tree dies. The question as to why trees do not grow into the sky, and why different species have different limits to their height growth is still open for speculation. We dispose of it here simply by referring it to internal disposition, in some degree modified by site conditions, for the maximum height which the same species attains varies with soil and climate. Some species, like the white birches and aspen, attain their maximum height early, but most of our timber trees reach it in 100 to 150 years. The tallest, such as the big trees of California and the eucalyptus of New Zealand with 300 to 350 feet, perhaps take a longer time to accomplish this height.

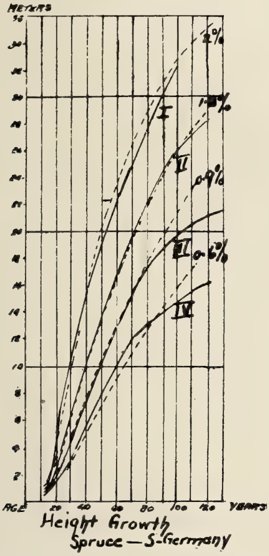
Merely to suggest a physical explanation of the limit to height growth we can imagine that gravity and friction, variable in different species, increase with the growing height for the transpiration current of water which is needed for carrying on life functions, until in drouthy years the supply of water cannot be secured by the leaves and buds and shoots at the tip fast enough, and they die back, to be replaced again and again by new shoots, but unable to overcome the limit set. According to well-known mechanical laws the lifted load of water must be inversely proportional to the heights to which it is elevated; hence in the same proportion as the lifting force decreases through friction and gravity, the annual increments must decrease; that is to say, the higher the tree, the slower its annual height increment. And, reasoning back from an observed or experimentally found maximum height, under the assumption that the lifting force remains the same, and that we have a measure of the initial force, we can construct the formula which expresses the curve of the gradual decrease in the rate and the height at any age (a)

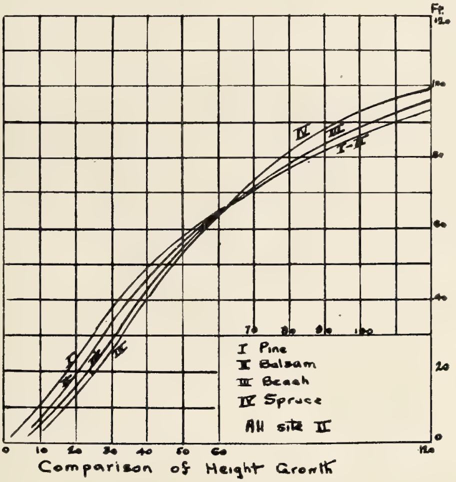
$$h_a = h_{max}\left(1 - \frac{1}{1.op^x}\right)$$

If we know, therefore, that a species attains a maximum height of 35 m; that the growing power of the site p is 2 per cent., we know that the height at 60 years would be $35\ (1 - \frac{1}{1.02^{60}} = 35 \times .69522 = 24.3327\ m$.

By actual measurements this formula or law of height growth has been found correct after the juvenile stage is passed. During this latter stage gravity plays a small role and internal disposition even more than external physical conditions, etc., has much influence; the rate during this period is very variable in different species, but follows mostly a compound interest series, i.e. an increasing rate. Some species like the beech and fir start very slowly into the arborescent form with a definite leader; they remain for a number of years shrub-like with bushy crowns, while others, like the aspen and elm, start at once with a rapid height growth, lacking entirely or almost the slow youthful period. It is this difference in the young stage that makes us talk of rapid and slow growers. But the forester who contemplates the future more than the present has learned that the law of the lever rules the world here too, and what is done quickly is usually not done persistently. As a rule the light-needing species are the quick growing in the early stages, but usually they are not persistent, and many, like the aspen and locust, remain small trees.

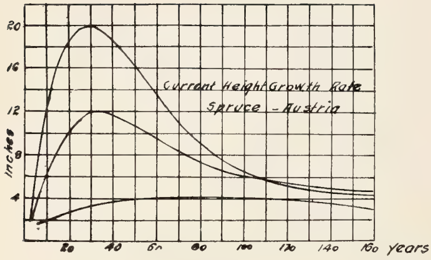
The tolerant or shade-enduring are mostly the slow but also persistent growers. With these the juvenile period may last 10 to 25 or even 30 years, while the light-needing rarely exceed 5 years before they begin to increase in height under the law of gravity and friction as elucidated. We find then generally a slow progress followed by a rapid increase in rate to a maximum which we may designate as the adolescent stage, and then the longer period of slowly decreasing growth and finally relative stability, which in the curve would be expressed by a horizontal or rather a wavy line of small extension.

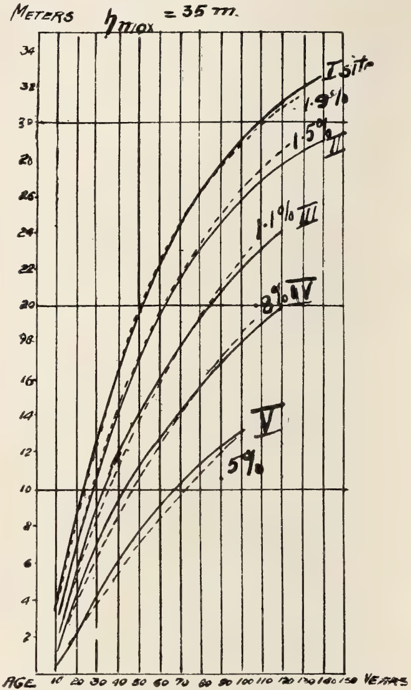




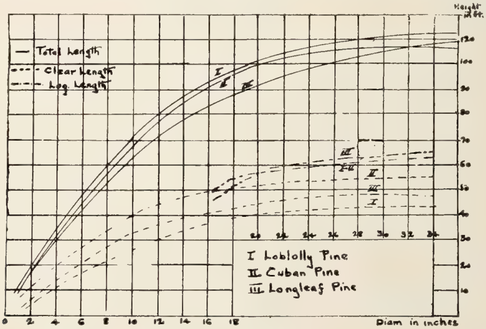
After the juvenile adolescent period is passed a remarkable uniformity of height growth among different species is, in most cases, noticeable, so that if the curves were shifted in such a manner as to exclude the time of the juvenile period, they would nearly cover each other. On good sites the rate p in the formula lies near 2 per cent. up to 2.5 and 3 per cent., while on poor sites it sinks below 1 per cent. It will be clear that the maximum annual or current increment lies somewhere at the beginning of the curve after the juvenile period. With light-needing species this occurs as a rule before the 20th or 25th year, with the shade-enduring species between the 30th and 40th year.

In discussing *diameter growth* the first question is where to measure it. The rule is to refer to the diameter at breast high —*d b h*—say 4 1-2 feet from the base, partly because of the convenient access to this point, partly because below this point (and in very old trees even above this point) there are found root swellings which, if measured in, would give an erroneous picture of facts. By locating the measurement at this height, we exclude at once in most cases the juvenile period from consideration, the time when the tree has still more or less shrub form, and even in the dense forest is more or less branched to its base; the time when the tree is growing into crown rather than into bole, and it is the bole alone that interests the forester. While during this early period the tree is putting vigor into height growth and branches, the diameter increase is very slow and its measurement of little practical value. The duration of this



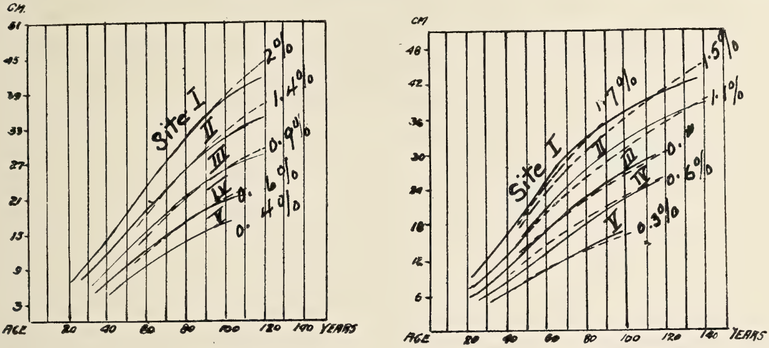


Height Growth
Pine — N. Germany



Comparison of Diameter Growths

juvenile stage, i.e. until the breast height diameter has reached say 6 inches and the bole up to that point and somewhat above is clear of branches, may last 15 to 20 years with light-needing, and 30 to 40 years with shade-enduring or slow-growing species. After this period of slow diameter growth the increment progresses for a long time at a rate which corresponds to the square root of a constantly rising multiple series of the form $d = \sqrt{\frac{4px}{\pi}}$ in which x, the factor time, represents age diminished by the juvenile period, $(a\text{-}i)$, and p is the experimentally found growth vigor of the site. The culmination of the diameter increment rate with most timber trees occurs between the 40 and 60 year on good sites, and some 10 to 20 years later on poor sites. The value of p on best sites lies between 1.5 and 2 per cent. with regular decreasing progression from the higher to the lower site classes.



Spruce . N. Germany

Pine N. Germany.

DIAMETER INCREMENT
AVERAGE TREE

Incidentally I may mention that to determine the rate of growth at which a tree is growing, all that is necessary is to secure a core or chip showing the growth of the last 10 or 20 years, when by the use of the formula $p = \frac{400}{nd}$ in which d is the diameter, n the number of years which it took to make one inch, the probable rate for the next 10 or 20 years may be determined.

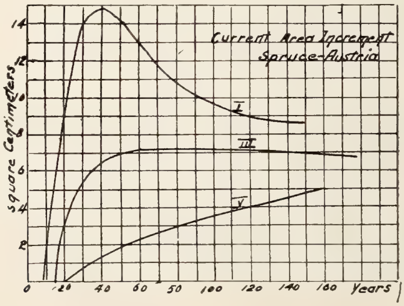
A quick mathematician will at once see that the formula for the diameter growth is derived from the more obvious one $\frac{1}{4}\pi d^2 = p\,x$, which expresses the relations of the basal area as a direct function of time.

To the forester who is concerned with volume production the area increase of the tree is more directly of interest than the diameter, although to be sure, he has to measure the latter in order to secure an insight into the area increment. That the area does not grow in the same proportion as the diameter but as the square of the radius every mathematician knows. Hence the width of the annual ring may be decreasing, yet, being laid on over a larger periphery, the narrower ring may represent a larger area; in other words, neither the diameter increments nor the width of annual rings give directly an insight into area or volume increments.

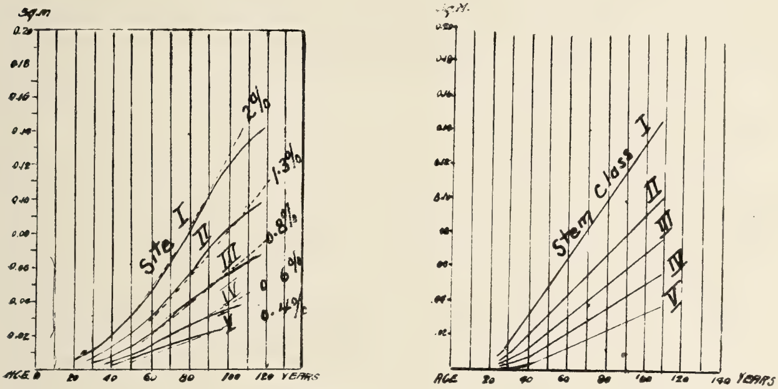
No other measurement expresses so completely the growth conditions of the tree as that of the cross-section or basal area; from it the life history of the tree can be most readily read.

Examining the cross-sections of various trees, we may find this one showing a fair ring width in the centre, which later rapidly declined; this attests that it was once among the dominant, but gradually fell into the sub-dominant. Another shows by the narrow rings in the centre that it started life with small chances, but the sudden change in the rate of growth tells us that an overshadowing neighbor or branch was broken or removed, changing the conditions of growth. Others again show by their fair and even progress that they grew up in close position, which permitted only slow but constant increment.

While in the forest the trees which are more or less deprived of light by neighbors, show naturally great variations in their area growth, and usually with age a rapid decline; the dominant trees or the main stand, in full enjoyment of light, show a most remarkable regularity in area increment from period to period, which, unlike the height and diameter growth, follows a simple multiple series of the form $A = p\,x$, i.e. the area is a simple function of time, the curve becomes a straight line; in other words the basal area increases after the juvenile period in the manner of a simple (not compound) interest series with the rate p. The value of p, the growth energy, varies in the forest not only with species and site, but is especially influenced by difference in light conditions which differently placed individuals enjoy. It is customary to classify trees from this point of view into five classes: the predominant and codominant having their



crowns free and not overshadowed by neighbors, the subdominant having their crowns already narrowed in, being in danger of falling into the dominated classes of oppressed and suppressed which are doomed to death by the shading influence of their neighbors. The values of *p* for dominant trees lie between 1 per cent. and 4 per cent., while the lower stem classes range below 1 per cent. E.g., in a spruce forest the growth energy of the different tree classes varies nearly as 1 : 2 : 3 : 6, from the suppressed to the predominant, because the first have only poorly developed crowns with few leaves in poor light, while the last have an abundance of foliage in full enjoyment of light and hence grow six times as fast. In other words, the basal area grows in proportion to the active foliage area. The maximum of the area increment occurs in trees grown in the open mostly long after the 100th year, in the dominant trees of the forest usually between the 50th and 80th year, but on poor sites much later.



SPRUCE N GERMANY

SPRUCE SITE II SAXONY

BASAL ACRE INCREMENT

AVERAGE TREE

An interesting biological explanation of the difference in behavior of different species as regards the absolute amount of basal area increase is found in the consideration that on a given acre with given growth conditions, with the same amount of sunshine and water, all species will produce the same amount in *weight* of vegetable substance. Hence, species with high specific weight, the hard heavy woods, must show smaller annual rings and area growth than the light soft woods, which for the same weight must deposit larger volumes of wood.

Before looking into the mathematics of volume growth, it is necessary to have some idea of the *form* or shape of the bole of trees, which means an insight into the diameter or area growth *at varying heights* outside the breast-high point of measurement.

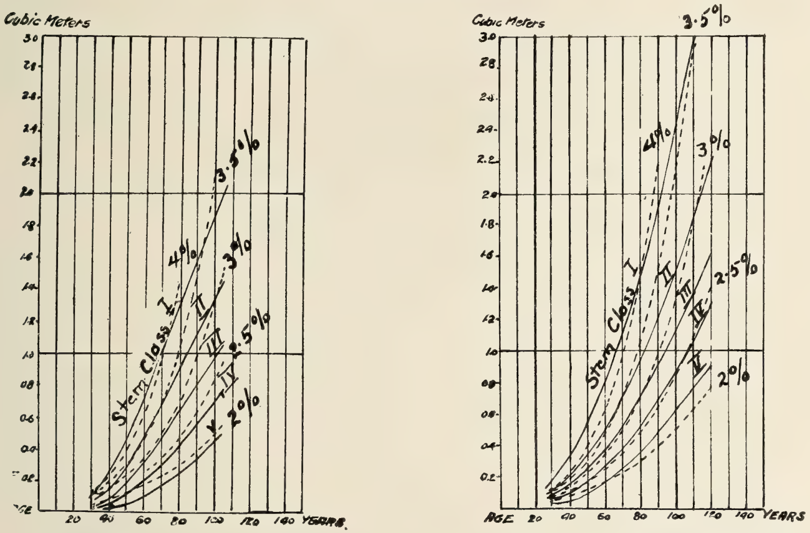
The form of a tree bole is neither a cylinder nor a cone, but it *tapers* unevenly and lies between the two, a paraboloid or neiloid, with variations which are mainly due and depend upon the character or amount of crown. The tree in the open, branched to the base, develops a form almost conical, the width of the same annual ring increases from top to base and so does the diameter in proportion, since each additional branch adds to the food supply. A tree with a small crown cannot elaborate food materials enough to make even a uniform envelope all the way down, the ring width and the area decrease from below the crown downward, and the result is a cylindrical trunk. The older the tree, even in the open, the more does this proportion of crown to circumference decline, and the more cylindrical does the form of the bole grow. In very old and poorly fed trees, the deposit of wood may fail altogether in the lower portion and the annual ring is not developed. In the crown the same conditions, an increase of ring width and area downward occurs, because each additional branch adds food supply, and hence the top becomes conical. Within these two extremes of nearly conical and nearly cylindrical bole, a number of other forms are found according to size and character of crown and species. By a large number of measurements so-called *form factors* have been established by which for different species, different sites, different age, and different phases of development, the variation from a cylinder is expressed, so that the volume of a tree is expressed by the formula $vol. = a.\ h.\ f.$

The values of f lie usually between .4 and .6, i.e., the volume of the bole of a forest grown tree is about one-half the volume of a cylinder with the basal area at breast height and the total height of the tree.

There have lately been other interesting mathematical relations worked out between the diameters at varying heights, which we cannot stop to discuss.

The *volume* increment is finally the most important, and naturally exhibits the resultant of all other directions of growth. Here the periodic stages of development or changes of rates are more evident than in all other directions of growth. During the juvenile stage the tree is concerned in developing its root system and the assimilating organs; there is little volume growth of the stem, so little indeed that it cannot be shown in the curves for the first two decades or more. The length of this slow juvenile period varies, as in all other directions, with the species, with the site, and with the density of the forest. The shade enduring species, as a rule, extend this slow period longer than the light-needing. A close stand, where the individual is curtailed in its opportunity for expansion, has the tendency to lengthen the slow period, a good site where trees are best fed, to shorten it, while open stand and poor site have the opposite effects. A most important deduction may be made at once from the realization of this slow period, namely that with shade-enduring species and on poor sites the rotation, that is the harvest time, must be longer deferred, else we repeat the slow period too often and the total production is curtailed merely by this repetition of the slow period.

After the 10th to 25th year when the trees are well established (and on poor sites much later), the volume increment starts on a rapid and constantly increasing rate as the concave curves show, namely at a compound interest rate of the form of an exponential series $y = 1.0p^{x} - 1$. Such a rate continues often for many decades and even a century or more, on the poorer sites, the white pine being perhaps the best exponent of the persistent growers. Here, too, we can again observe the law of the lever: The greater the *p*, i.e. the more favorable the growth conditions of the site, the sooner comes the decline to a lower rate, until the compound interest may fall to a simple interest series; the curve becomes a straight line. On the other hand when the initial rate is slow, it remains constant the longer. The maximum of the annual rate lies towards the end of this ascending period and does not consist in a single culminating year but remains for several years constant before the declining rate begins, i.e. before compound interest rate changes to simple



SPRUCE–II SITE–SAXONY SPRUCE–I–SITE–SAXONY

VOLUMNE INCREMENT

interest rate. This with many species takes place so late in life that the inquiry loses its economic value: the trees are harvested before that time, for value increment per unit ceases to grow before volume increment declines. All other things being equal, the amount of light at the disposal of different individuals makes the greatest difference in their increment and hence the density of the stand, the number of trees on an acre, influences the absolute amounts of wood deposited on the different individuals.

While, therefore, for dominant trees the value of *p* lies between 2.5 and 6 per cent., the third class stems would be reduced to 1.8 to 4 per cent., and the lowest class remains somewhere below 2 per cent. The rates of volume growth for the five stem classes, from the lowest to the dominant, progress as 4, 5, 6, 7, 8, that is to say, the best placed trees will grow twice as fast as the poorest placed, and sometimes the difference is even greater. But, while the rate in the dominant reaches its culmination early and then declines, the low rate of the slower growing tree may continue for a much longer time at even progress.

Similar differences of behavior are found with trees of the same species on different sites. For example, the Norway Spruce on best sites shows culmination of the rate of volume growth between the 60 and 80 years with 8 cubic feet, then declining slowly. But the poorer sites have not reached the end of the increase in rate in the 160 year, and often continue to increase beyond the 300 and 350 year, although, of course, the actual rate remains much lower during the whole life than on the good site.

It appears then, that in all directions, in height, diameter, and volume increase the greatest growth energy is developed on good sites in the younger years or middle life, with a subsequent decline, while on the poorer sites the growth energy is slow but steady for a longer time.

By proper management of the light conditions the forester has it in his power to influence the rate of growth and to make even slow growing species produce more per acre than they naturally would. To give a concrete example of how this volume increment progresses, we may refer to the Scotch Pine, probably closely resembling the Red or Norway Pine which from 10th to 80th year grows on best sites at the rate of 3.5 per cent.; from the 80th to 110th at 3 per cent., but on third class sites, which are its common heritage, the rate is only 2.5 and 2.3 per cent. respectively.

ACRE INCREMENT.

The forester is, after all, only incidentally interested in the single tree, he deals with acres of trees, and while the single component has, perhaps more economic importance to him than the single blade of wheat to the farmer, the acre production must finally be his main interest.

Lawless as the mixed forest appears to the layman, the biologist knows how to account for the existence of each component, and can analyze the laws under which the composition has come about, and how it progresses in development. Hopeless as the task may appear, the forester has learned to measure the amount of progress, how much the performance from period to period has been and is likely to be under similar conditions.

If it is necessary to classify carefully and generalize broadly in defining the law of progress of single individuals, the classification must be still more circumspect where so many variables are possible, and the generalizations must be drawn still broader with acres. The first broad generalization which a mere biological consideration will lead us to, and which measurements have proved to be correct is this: that an acre of given quality will produce, with given sunshine and rain, the same amount of dry weight of vegetable matter without regard to the species employed. This vegetable matter, to be sure, may consist of roots, stems, branches, leaves and fruit in varying proportions, and in volume, especially the useful volume, the differences may be very great according to the looser or denser structure or the varying specific weight of the plants. But if the consideration is confined to the economically valuable woodgrowth above ground, it may be stated that timber trees which form stands produce approximately the same amount of wood in dry weight, namely annually between 2,700 and 3,600 lbs. per acre on first class sites, down to 1,350 lbs. on poorest sites. Knowing the specific weight of the different species we can then arrive at the volume production by multiplying the weight per acre with the specific weight of the wood of the species, and may learn that one species e. g. White Pine, may produce from the same acre 3 to 5 times the volume which another species e. g. the heavy beech, is capable of producing.

It is also of interest to note that although great changes in the number of trees per acre occur, the *annual* product of wood in weight inclusive of bark continues constant for a long time until high age, with only small variations.

The important deduction is to be made from this that not the number of individuals, but the closeness of the crown cover, i.e., the amount of foliage at work, will decide the volume product per acre, hence that the increment can by thinning be transferred from one tree class to another, if this is capable of growing into the space opened up to it.

In a given case the number per acre had been reduced in 100 years from 16,000 to 112, thinning out and removing from time to time very considerable quantities of wood, without decreasing the average of total weight increment per acre from decade to decade.

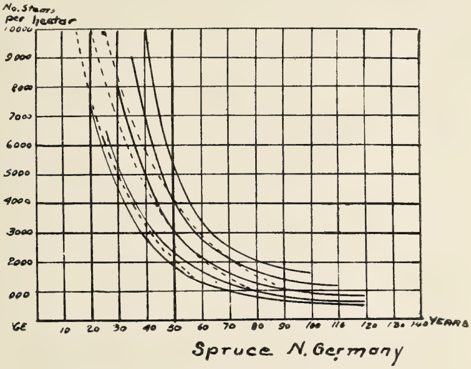
The specific weight of wood itself really decreases with age, that is to say, young wood is heavier than old, so that this compensates for the small volume production in the juvenile and the increasing rate in the long virile period: the volumes are inversely proportional to the specific weights: $V_1 : V_2 = \frac{1}{S_1} \frac{1}{S_2}$ This holds not only from species to species, but from individual to individual.

There are three considerations always present in the forester's mind when he discusses density of his crop, i. e., tree numbers per acre. A dense stand produces branchless boles, a desirable form: it also makes a close crown cover, shades the ground, an essential factor in keeping soil conditions favorable. On the other hand, with many trees to the acre the wood production is distributed over so many individuals that each secures but little, even the best grow only slowly in size, and size is an important quality for the use in the arts, hence the problem is how far to follow each of the three aims, of form, of soil cover and of volume increment to be deposited on as few individuals as will suffice to satisfy the first two objects.

It is evident that the question of numbers is of utmost importance. In the dense sowing of nature a quarter million plants may start together on an acre, but in the first years the infantile death rate, due to lack of vitality and unfortunate domestic conditions, as in the human world, is great, and probably not over 100,000 live through the first year. Then, as these expand with their roots into the ground and with their foliage into the air space, there comes competition for space, for water, for food materials, for light—a struggle for existence. Some more favorably placed or of more rapid height growth overtop others, and by and by a differentiation into tree classes is discernable, such as we have alluded to: there are developed masters or dominant, and subordinates or dominated. The latter are gradually crowded out of the race for lack of light, or water, and are suppressed and finally die. Just as in the single tree, of the many buds which are set only a small percentage can develop, so on the acre there is with the expansion of the single individuals a reduction of space and this can be met only by reduction of numbers. This reduction, whenever the stand is a close one proceeds from year to year or from decade to decade at a certain rate, which, like all performances in nature, varying of course with species, sites and age, can be nevertheless demonstrated as subject to certain laws. It is clear that this decimation must proceed in inverse proportion to growth, and hence to the shading value of the dominant class, and that means mainly in proportion to height growth, so that a curve for the decimation in numbers, or death rate, would appear as the negative picture of a height curve.

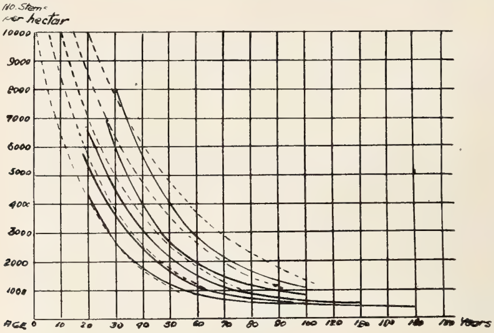
After the juvenile period, which may last from 5 to 35 or even 50 years, and when the average standing room per tree has become about 10 square feet, or when say 4,000 trees occupy the acre, and the rapid height growth begins, the struggle becomes fiercest. As long as the height growth is rapid and inrceases, as we have seen, at a compound interest series the numbers must also decrease as rapidly in a reciprocal or discount series of the form $\frac{1}{1.op^x}$, or by introducing the acre figure $\frac{43560}{1.op^x}$ will be the number at any time for this period; x being the factor time or age minus juvenile period. Then, when the rapid height growth period is passed the decimation slows down to a simple discount series of the form $\frac{1}{p^x}$. And, when the height growth is nearly finished and the number has sunk to about 400, the rate is further reduced to $\frac{1}{1.op^{\frac{x}{2}}}$. Finally, when about two hundred trees occupy the ground they can peacefully live together without interference, and only accidents diminish their number, so that only 1-10 to 1-50 of the trees that started to grow will be found at harvest time on the same acre.

On good soils and with rapid growers, when dominance is early developed, the numbers fall more rapidly than under opposite conditions. In other words, there are more individuals at a given age to be found on poor sites than on good, and the difference in development of single individuals of the different tree classes is not so pronounced as on good sites. The values of p vary from 7 per cent. to 8 per cent. for spruce and fir on best sites, down to 3 or 2.5 per cent. for pine on poorest sites.



NUMBER DECREASE PER HECTAR

As stated before, the forester interferes with the natural progress of decimation by thinnings. Having first secured by dense position a branchless bole of sufficient height he secures a more favorable distribution of growth on fewer individuals, thereby hastening the development of the fewer select or *elite* to serviceable size.

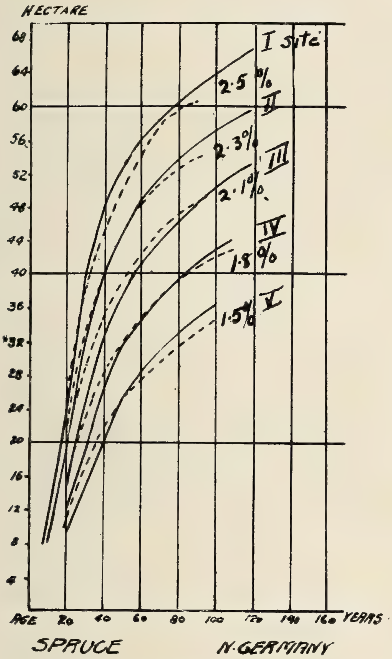


Number Decrease per hectar
Pine S. Germany.

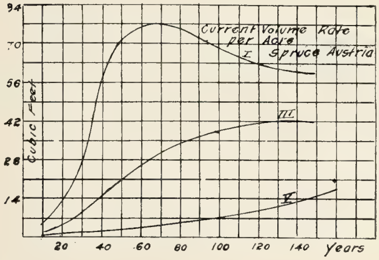
Now, to secure a conception of how the volume production of a whole stand per acre progresses we must first inquire into the factors which produce the volume, namely the basal area of all the trees and their average height and form. We can imagine the basal or cross-section areas of all the trees combined or summed up into one area, the basal area of the stand, and consider the changes which this total area undergoes through the life of the stand. There is an increase in the basal area of each component and hence of the total or sum of all, but there is also a dropping out of individuals and this influences the rate at which the total basal area of the acre progresses from decade to decade. Instead of using the sum total, we may divide it at once by the number of trees involved in each period, and thus secure the progress of the average basal area or the area of the average tree.

During the juvenile period the competition is not keen, and often young stands start without their full complement of plants, the stand is open, and no lawful number decrease takes place. Only when the crown cover is closed, does the competition begin and the two factors of individual increase and of number decrease enter into the problem. This juvenile stage as a rule does not last as long as when the single individual is only concerned. With rapidly growing trees 5 to 10 years, with slower growing 10 to 15 years and rarely more, brings the stand into a well-defined law of progress. It is evident that if we combine the formula of number decrease and of the average tree increase into one expression we must get the basal area increment for the whole stand.

Such a combination brings us, indeed, to the experimentally established formula of the curve, which during the period of rapid decimation reads $2\ A = \frac{px}{1.op^x}$. In other words both this theoretical development and the actual measurements show that the higher the growth energy of a stand, the more rapidly, to be sure, increases the area of the single stem, but also the more rapidly sinks the stem number, the former following a multiple series of the squares of p, the latter following the reciprocal of an exponential series with the basis $1.op$, and the two items stand in the relation as expressed in the formula.



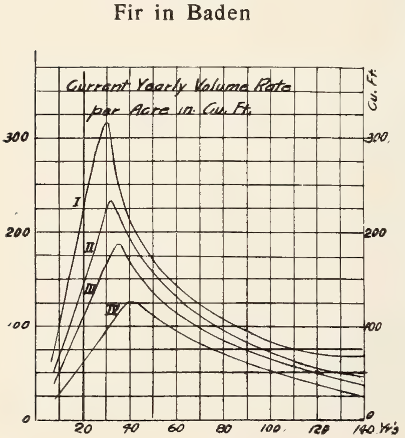
BASAL AREA PER HECTARE

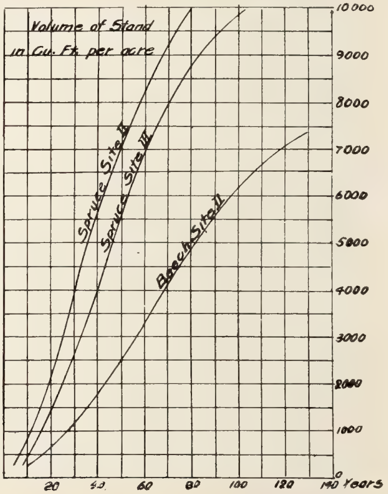


The culmination of this series is reached when the death rate changes, and of course the sooner the greater the growth energy; the higher the p. Then follows a period, during which the basal area grows still constantly but the numbers decrease more slowly, requiring twice the time for the same amount of reduction, the denominator of the formula becoming $1.op^{\frac{x}{2}}$, that is to say, the curve becomes shallower. For different species and sites the values of p vary between 1 and 2.8 per cent., the shade enduring conifers and the best sites making the best progress.

Finally we come to the volume increment per acre, the ultimate object of the forester's work. The volume per acre is of course the product of the number of trees and the volume of the average tree, or from what has been shown before for these two factors, the increment is expressed by a combination of the two series of the form $\frac{1}{1.op^x}$ and $1.op^x - 1$, that is, $vol_x = 1 - \frac{1}{1.op^x}$ This expression we find is analogous to the one found for the height curve of the single tree, and so we come to the realization that there must exist proportionately between the progress of height growth of the average tree and the volume increment of a stand. This interesting fact of the analogy of height growth and volume growth of closed stands explains itself readily by the consideration that the number decrease is a consequence of the height growth of the dominant stem classes which suppress their weaker brethren. Both, decrease of height growth and decrease of numbers progress according to the same reciprocal series, namely $1 - \frac{1}{1.op^x}$ and have therefore the same compensating influence on the increment, which would otherwise progress according to the compound interest series $1.op^x - 1$.

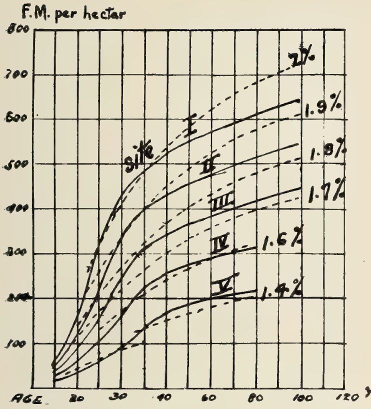
In other words, the limit which the soil area sets to expansion has the same mathematical influence on the total volume increment, as the influence of gravity has on height increment. Or, again, the increment of the average tree is inversely proportional to the number diminution, since with increased p the volume of the single tree grows faster, but also the number of



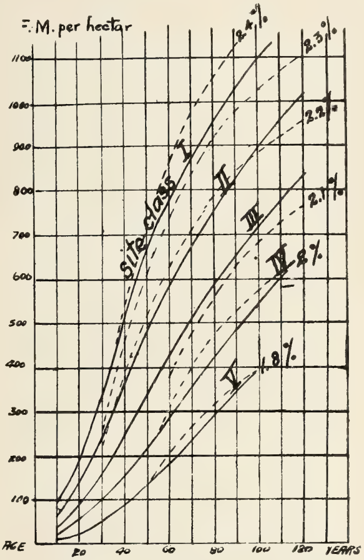


trees decreases faster. To the experienced forester the height growth becomes therefore the best measure of the site quality. Especially the depth of the soil finds very striking expression in the character of height growth and crown development.

It is characteristic of the current volume increment rate per acre that on the better sites a very decided culmination of the rate occurs the earlier the greater the growth energy, and that to the rapid increase in rate of the time before the culmination corresponds an equally rapid decline after it. In spruce the culmination may occur on best sites around the 50 year with say 250 cubic feet per acre, in medium sites some 15 years later with only 115 cubic feet, and on poor sites with less than 50 feet after the 100 year.



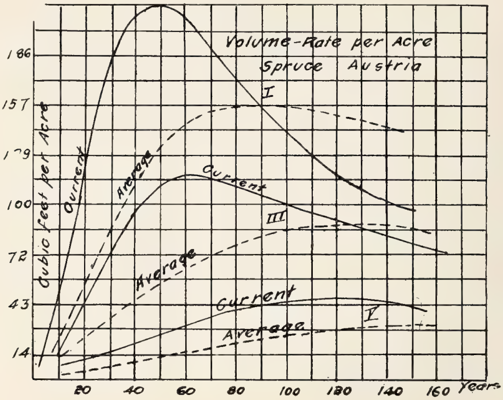
Volume Increment per hectar
Pine Saxony.

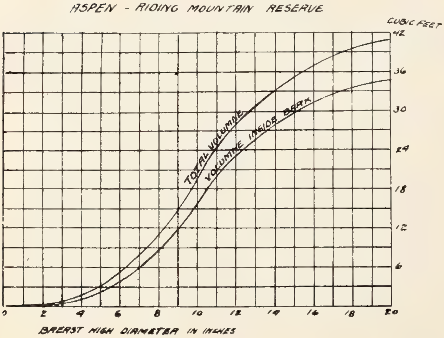


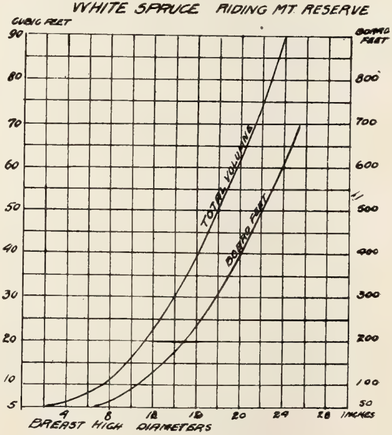
Volume Increment per hectar
Spruce S. Germany.

On the whole the progress of volume development of stands proceeds more slowly than that of single trees, the stem number furnishing the moderating influence.

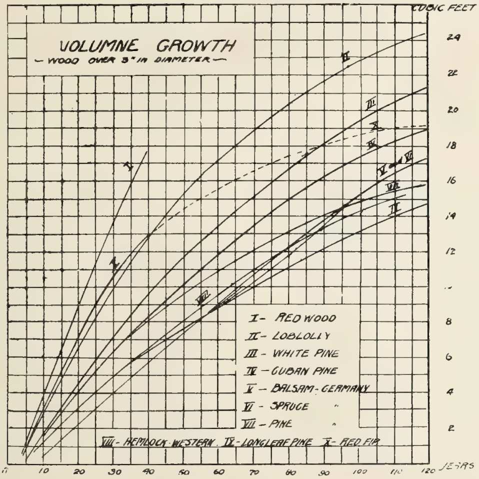
One of the important questions which the forest manager asks is, not when the highest rate of growth occurs, but when is the maximum production per unit of time attained; when is the highest *average* increment reached? It is evident that this has occurred when $\frac{vol.}{time} = maximum$, and it can be mathematically proved by differential calculus or by mere logical consideration that this occurs when the average increment is equal to the current increment. Contrary to former beliefs, this time of maximum volume production occurs rather earlier than was supposed, usually with light-needing species, which have a short juvenile period and rapid development, before the 40 year, with shade-enduring before the 80 to 90 year, on better sites earlier than on unfavorable ones. Since at those ages most trees are still too small for advantageous use, it cannot be the aim of the forester to grow the largest amount of wood per acre, but in order to determine the proper harvest time, the further modification of quality and value increment must be made, which can also be determined by mathematical methods.







There is only time left to present a few striking deductions from the yield tables. So lawfully does a stand of white pine develop, if naturally grown, that when its height growth is practically finished we can determine its contents by measuring merely the three stoutest and seven of the slimmest trees, and dividing by ten when we have the volume of the average tree which multiplied by the number of trees making up the stand, gives us the volume of the stand. Again, by measuring the diameter of all the trees and ranging them in sequence by size from the stoutest to the slimmest the average tree will be found at 40 per cent. from the stoutest. We can also feel certain that in a pine forest grown naturally, if the trees be arranged according to diameter classes, 40 per cent. of the total volume of the stand is represented by the first 20 per cent. of trees; 24 per cent. by the second 20 per cent., 17 per cent. by the third, 12 per cent. by the fourth and 7 per cent. by the last 20 per cent. In other words, the stoutest half of the number of trees represents more than two-thirds to three-fourths of the whole volume.



I hope to have impressed you with a realization that even the least promising conditions of nature can be reduced to mathematical expressions and can be proved to be the result of lawful procedure. Some time, we hope, instead of relying on European species and conditions to prove these laws as applicable to our own species and conditions, when we shall find that nature works on the same lines on this continent as in the Old World, and that, whatever difference in economic conditions may exist, the silviculture of Europe is applicable here.

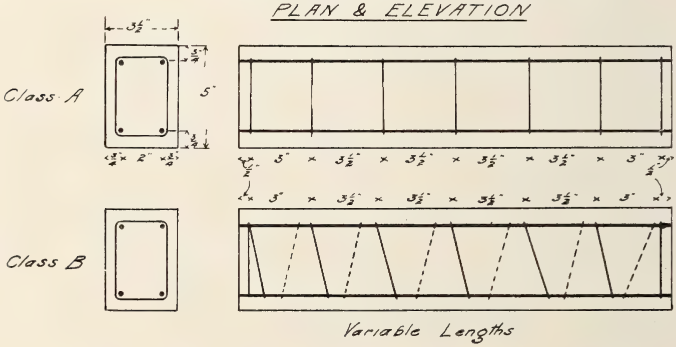
REINFORCED CONCRETE COLUMNS

P. GILLESPIE, B.A.Sc.
Lecturer in Theory of Construction
W. G. SWAN, B.A Sc.
Demonstrator in Applied Mathematics

Perhaps no subject in the engineering world has excited more interest or comment within recent years than that of reinforced concrete. Its comparative cheapness, its adaptability to so many varieties of construction and the general satisfaction which it has given when the design has been careful and the work reliable, have given it ever increasing popularity. The intention of this article is to discuss briefly in the light of some recent experiments, the subject of reinforced concrete columns, these experiments having been conducted in the testing laboratory of the Department of Engineering. We are indebted to a large extent for these results to the Roman Stone Company of Toronto and to Mr. G. B. Ashcroft, its former superintendent, through whose kindness the columns in question were made and presented to the Department. With the outlook for better facilities in the near future, in the department of Applied Mechanics, it is the intention to pursue carefully the study of this and allied problems.

These reinforced columns were of two varieties: Class A, in which the upright reinforcement consisted of four 3-8″ round steel rods placed as shown in the sketch with an additional rectangular hooping of 3-16″ steel rods; Class B, in which the upright reinforcement was similar to that of Class A but where a spiral form of 3-16″ reinforcement replaced the rectangular form of the other series. The following table gives the dimensions of the various columns tested:

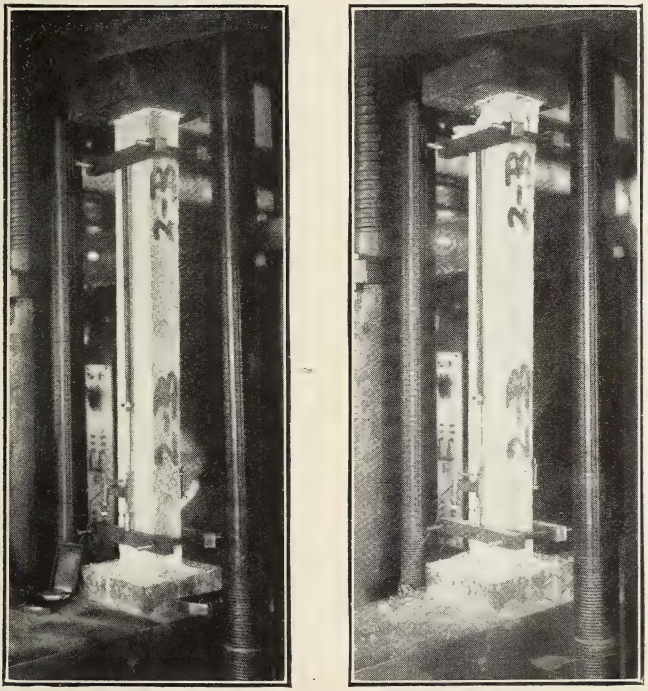
REINFORCED TEST COLUMNS
PLAN & ELEVATION



Reinforced columns:
- Column 1-B, 3½"x5"x21" in length.
- Columns 2-A and 2-B, 3½"x5"x42" in length.
- Column 3-B, 3½"x5"x63" in length.
- Column 4-A, 3½"x5"x84" in length.

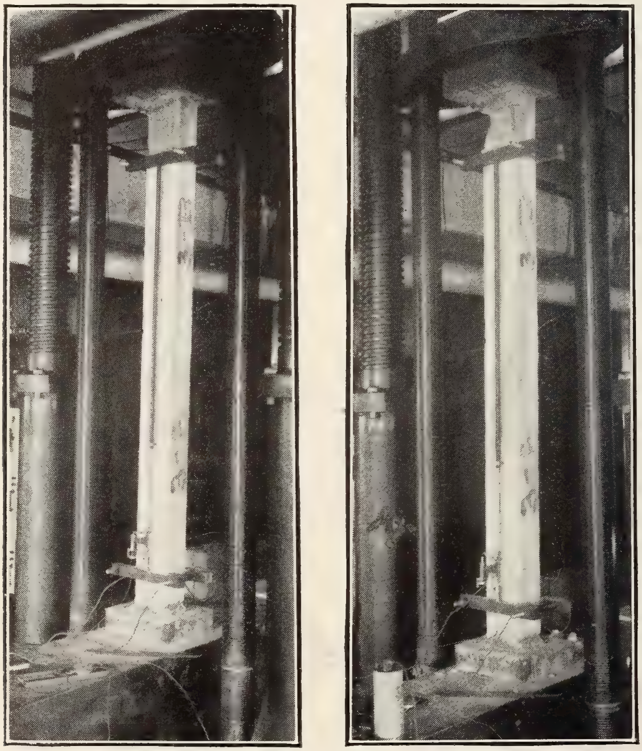
Non-Reinforced columns:
- Column No. 1, 3½"x5"x20" in length.
- Column No. 2, 3½"x5"x23" in length.

The columns were tested in the 100-ton Riehle' machine of the testing laboratory; they were set carefully in place so as to be truly plumb and each end was well cushioned against the cast iron heads of the machine by the use of mortar. A mortar of neat cement was found to be most satisfactory provided the columns were allowed to stand over night before testing, in order to give time for the setting of the cement. Plaster of paris was used as cushioning mortar in the testing of columns 2-B and 4-A but its rapidity in setting made it difficult to handle and did not permit of an even setting of the ends of the columns. It was also noted that the ultimate load borne by these two columns was considerably below the average.



Column 2B Before and After Crushing

The deformations due to loading were measured by means of the apparatus attached to the columns as shown in the half-tones. This apparatus consisted firstly of two collars attached to the column not far from its upper and lower extremities. To the upper collar and on opposite sides of it were made fast two telescopic rods which could be adjusted for a varied length of specimen. A pair of micrometer screws obtained from a dismantled Reihle' compressometer were rigidly attached to the lower collars and on the movable arms of these were made to bear the telescopic rods. The loading of the column thus caused the collars to approach each other. These in turn caused the telescopic rods to force down the movable arms of the compressometer. The amount of this motion was registered by means of an electric contact device. The readings were recorded to $\frac{1}{10{,}000}$ part of an inch, the average of the two compressometer readings being taken to represent the true deformation. The loads were applied at intervals of 2,000 pounds and the compressometer reading noted for each. It will be interesting to note that in



Column 3B Before and After Crushing

each case the column failed in pure compression, no buckling of any kind such as usually accompanies the failure of columns of steel or wood having occurred.

The concrete was of a fine aggregate and was cast in horizontal sand moulds, as is the custom in the manufacture of Roman stone. An examination of the broken specimens failed to show any settlement of the aggregate at the bottom of the moulds so that the columns were quite homogeneous in structure. The concrete was matured in sand for a time and afterwards in air under cover so that practically all moisture was excluded. The average crushing strength, found from a number of tests made on samples of this concrete, was 1720 pds. per sq. in. German Dykerhoff cement was used. It may be of interest to note that the crushing strength of these samples is considerably less than has been found for samples of similar age taken from other runs of stone manufactured by this company and tested in the engineering laboratory.

The reinforcement consisted of mild open-hearth steel, purchased in the open market. All rods were round. The elastic limit was a trifle high but otherwise it was comparable with what is usually sold under that name. A table showing the properties revealed in some six individual tests is given below:

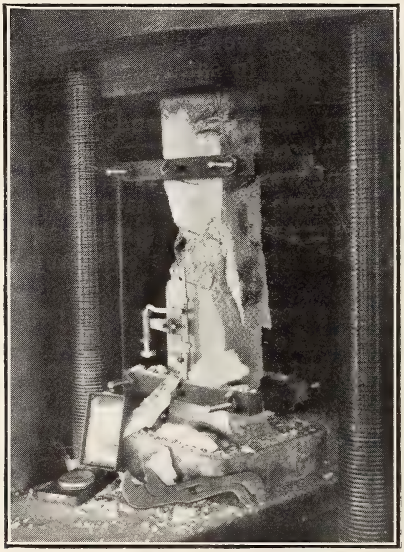
COMMERCIAL TESTS OF STEEL REINFORCEMENT.

Number of Specimen	Elastic Limit Strength (Pds per sq. in.)	Ultimate Strength (Pds per sq. in.)	% Elongation in 8 ins.	% Contrac. in Area.
1	43,100	66,600	25	57
2	41,900	64,800	26	59
3	43,000	64,600	25	56
4	43,100	67,400	27	55
5	43,100	67,600	27	53
6	42,900	67,500	26	54
Average	42,800	66,400	26	56

Figures 1, 2, 3, 4, 5, 6 and 7 are the stress-strain diagrams for the seven different columns described above. The ordinates represent stress, that is, the quotient of the total load on the column divided by the total area of the cross-section; and the abscissae represent the strains or the deformation per unit of length intercepted between the collars of the compressometer device. The absence of a constant ratio between stress and strain is apparent in every case, the curves deflecting downward in most cases almost from the start. This is in general agreement with what is frequently called the parabolic stress-strain relation and is in accord with the findings of all experimenters in plain and reinforced concrete.

In order to find what portion of any stress was carried by the concrete and what portion by the steel, a method adopted by Talbot was employed. It was assumed that the steel and the concrete deformed together. If the elastic limit of the steel be not exceeded, the stress therein will be proportional to the strain.

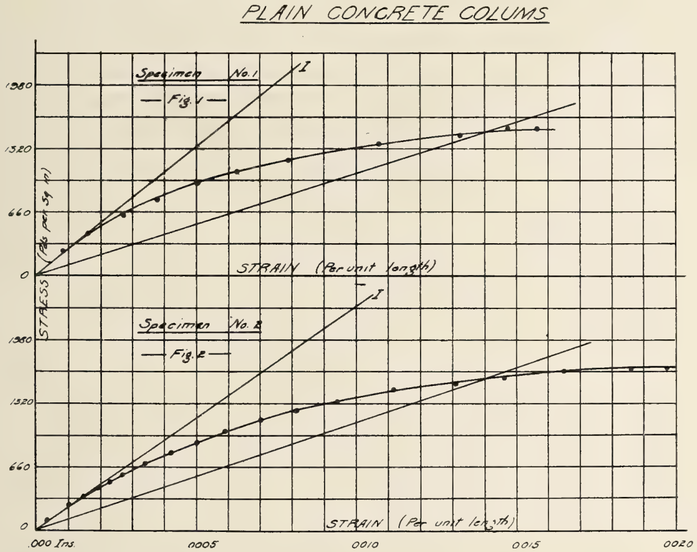
For a strain of .0014 say, the stress in the steel, taking E_s, Young's Modulus, to be 30,000,000, would be $f = .0014 \times 30{,}000{,}000 = 42{,}000$ pds. per sq. in. This, it will be observed, is a stress slightly below the average elastic limit of the steel as given in the table of results of tests thereon. Let A denote the total area of the column cross-section and p the ratio between the steel and the concrete. Then the total load carried by the steel, corresponding to the stress assumed, will be 42,000 pA pds. If this quantity be divided by the total area of the cross-section, A, we have as quotient, the number of pounds per sq. in. *of total area* which the reinforcement carries. The numerical value of this will be $42{,}000\, p = 42{,}000 \times \frac{2\frac{1}{2}}{100} = 1050$ pds. per sq. in. Through a point in the diagram whose co-ordinates are 1050, and .0014, a straight line was drawn to the origin. This will be subsequently referred to as the line for steel. From what has been said, it is apparent that the portion of any ordinate below this line will give the stress in pds. per sq. in., *based on total area of cross - section* which the steel supports. In consequence, therefore, that portion of the total ordinate intercepted between the line for steel drawn as described, and the curve as plotted, will be the stress in the concrete for the corresponding strain. Referring to figure 3, we see that the stress, 1620 pds. per sq. in. corresponds to a strain of .001. The line for steel shows that 750 pds. per sq. in. of this is supported by the reinforcement. From this it follows that the accompanying stress in the concrete



Column 1B Showing Mode of Failure

is the difference or 870 pds. per sq. in. The line for steel has been drawn, it will be noticed, in each case to the ordinate through .0014 as this corresponds to a stress of 42,000 pds. per sq. in., substantially equal to the elastic limit of the steel.

In order to find the maximum stress in the concrete *occurring either prior to or in coincidence with the elastic limit of the steel,* a tangent to the curve parallel to the line for steel was drawn. The point of tangency on the diagrams is denoted in each case by the letter *T*. Singularly enough, this point corresponds in every case very nearly to the elastic strain of the steel, viz., .0014. The values of the simultaneous stresses are given in the subjoined table. The stresses in all cases were scaled from the plot and the tangent was drawn by the usual geometrical method.

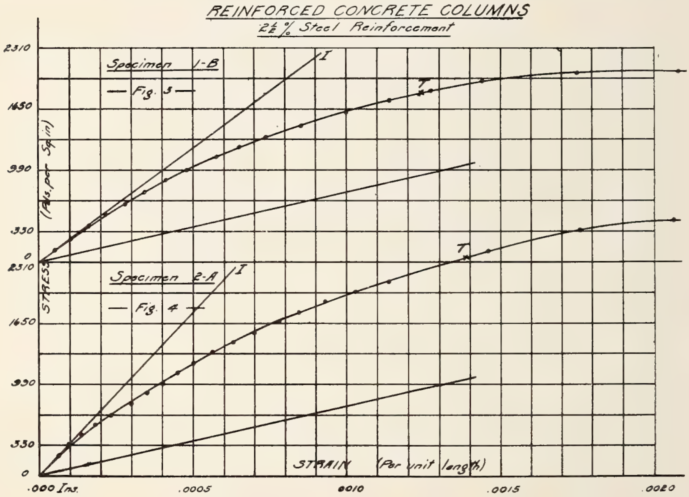

PLAIN CONCRETE COLUMS

Stresses Corresponding to Point T

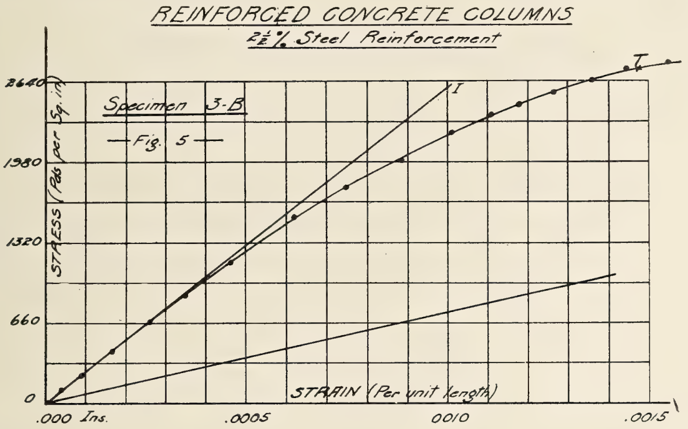
Column	Stress in Concrete	Stress in Steel	Strain
1 B	890	37,500	.00125
2 A	1,320	42,000	.00140
3 B	1,650	43,500	.00145
2 B	1,050	33,000	.00110
4 A	1,380	37,500	.00125
Average—	1,260	39,000	.00130

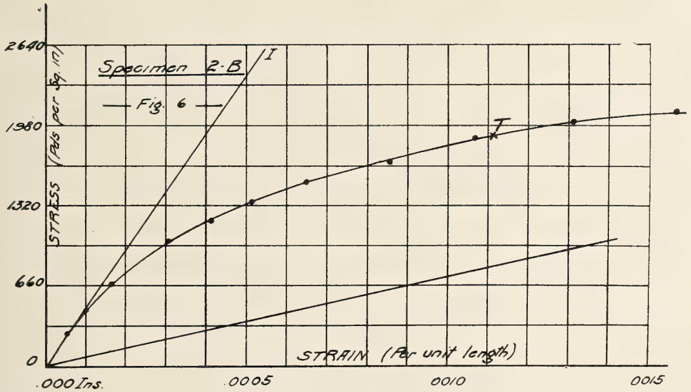
As to the manner of the distribution of the total stress between steel and concrete, after the deformation .0014 is passed, little can be said with certainty. The probability is, however, that the curve for steel flattens quite markedly, producing as will be seen from an examination of any of the graphs submitted, stresses in the concrete greater than those in the above table. This is within the realm of probability, since from the table given, it is seen that the average stress up to or coinciding with the elastic limit is only 1,260 pds. per sq. in. Now the average ultimate compressive strength of the concrete, it will be remembered, was 1,720 pds. per sq. in. so that a margin for increase remains. The rising of the stress-strain curve after the elastic limit of the steel has been passed, is thus capable of an explanation, although an exact analysis of these later stresses is quite impossible. The assumption has been made in this discussion, that the elastic limit in compression is the same as that in tension. The error in this assumption is probably very small.

If concrete hardens in air, a shrinkage occurs; if in water, an expansion results. Experiment has shown that a 1:3 plain mortar hardened in air will shrink, in consequence, from 1-20 to 4-20 of one per cent. in its dimensions. Where reinforcement is used, the deformations due to this cause are much less, but notwithstanding their smaller magnitudes they occasion compressive stresses in the reinforcement. A perfect bond between the metal and the concrete is of course assumed. Considere cites a case where a 1:3 mortar reinforced with 5½ per cent. of steel sustained a shrinkage of 1-100 of one per cent. in its length. This would occasion an initial compressive stress in the metal of approximately 3,000 pds. per sq. in. There can be little doubt that such initial stresses do exist but as to their actual magnitude, little with certainty is known. While their presence in this instance has not been wholly ignored, no attempt to consider their magnitude and effect has been made.



As stated above, the features of the stress-strain curve for concrete as obtained by compressometer tests is the variation of its inclination to the axis of strain. This is interpreted as meaning that the material loses rigidity as the stress is increased and that the so-called modulus of elasticity meanwhile grows less. It was thought worth while, in order to get a comparison between working stresses and factors of safety in the two materials, to find for a working stress of 500 pds. per sq. in. in the concrete, the modulus of elasticity of that material. This was done by finding with a pair of dividers, that point on the diagram where the vertical intercept between the line for steel and the plotted curve, was the graphical equivalent of 500 pds. per sq. in. The corresponding strain was noted and from the equation, $E_c = c/s$ where E_c is the modulus of elasticity for concrete, c the stress, and s the strain, the modulus of elasticity was found. This quotient of course will give what might be called the *average* modulus of elasticity covering a range of stresses from zero up to 500 pds. per sq. in. The corresponding stress in the steel was found from the equation $f = E_s s$, E_s being taken as 30,000,000.



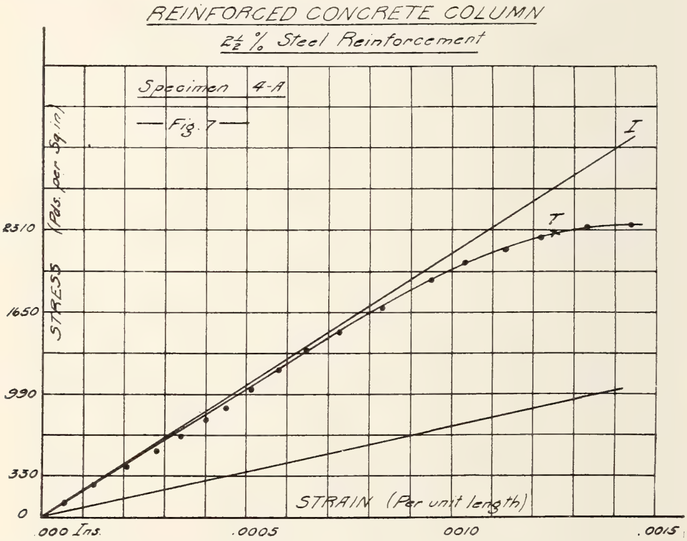


The factors of safety given in the following table are based on an ultimate crushing strength of concrete equal to 1,720 pds. per sq. in. and an ultimate strength in steel of 66,400 pds. per sq. in.

Moduli of Elasticity of Concrete Corresponding to a Working Stress of 500 pds. per sq. in.

Column	E_c	Stress in Concrete	Strain	Stress in Steel	Safety Factor-Concrete	Safety Factor-Steel
1 B	1,400,000	500	.00036	10,800	3.4	6.1
2 A	1,900,000	500	.00026	7,800	3.4	8.5
3 B	1,700,000	500	.00029	8,700	3.4	7.6
4 A	1,360,000	500	.00037	11,100	3.4	6.0
Average	1,590,000	500	.00032	9,600	3.4	7.0

The anomaly from the above table is that at such a moderate stress as 500 pds. per sq. in. in the concrete, the factor of safety should be nearly twice as large in the steel as in the concrete.



For this, no very satisfactory remedy seems to offer itself. There certainly could be no valid objection to utilizing stresses, in such steel as here employed, up to 16,000 pds. per sq. in. But unless we care to raise the working stress in the concrete above 500 pds. per sq. in. we must be satisfied with low working stresses in the steel. The stresses manifestly will always be proportional to the moduli of elasticity of the two materials. Evidently the most economical combination would be the somewhat unusual one of high working stresses in the concrete, with low modulus of elasticity. This would tend to an equality in the safety factors of the two materials.

It will also be noted on the diagram that a tangent line has been drawn in each case to the initial direction of the curve. This line represents the initial modulus of the material. The initial modulus of the concrete can now be found by eliminating the value for the steel, according to Talbot's method as previously mentioned. This initial modulus is sometimes employed in theoretic investigations since it corresponds to working values, which are the maximum values with the usual safety factor employed. The following are the values of the initial modulus of concrete for each of the columns tested. There appears to be considerable variation but the average value compares favorably with published determinations.

Column	E_c (Initial Modulus)
1 B	1,710,000
2 A	2,680,000
2 B	3,800,000
3 B	1,840,000
4 A	1,400,000
No. 1	2,630,000
No. 2	2,340,000
Average	2,350,000

It will be noticed that the points designated *T* on the graphs correspond fairly well with the strain at the elastic limit of the metal, viz., .0014. The modulus of elasticity for the concrete was found for this point also, the method adopted being that previously described. The results are contained in the table below:

Modulus of Elasticity of Concrete at the point T.

Column.	E_c
1 B	710,000
2 A	940,000
3 B	1,100,000
2 B	1,000,000
4 A	1,080,000
Average	966,000

Evidently, for stresses accompanying or approaching the elastic limit of the steel, an approximate value for E_c of 1,000,000 may be taken. This will render *n*, the ratio $\frac{E_s}{E_c}$ equal to 30 if E_s, be taken equal to 30,000,000 as above.

Now where a column is reinforced longitudinally, it can be

readily shown that the strength per unit area is given by the equation $c' = c\ [1 + (n - 1)\ p]$
where c' is the stress based on total cross-section,
c is the stress in the concrete,
n is the ratio $\frac{E_s}{E_c}$ for the stress c and
p is the ratio of metal to concrete.

This equation supposes that the two materials deform together and that the concrete adheres perfectly to the steel. The greatest stress in steel to which in fairness, this equation might apply, will be the elastic limit here taken as 42,000 pds, per sq. in., and for this stress, E_c was found to be approximately 1,000,000. The stress in the concrete for such a strain (.0014) will then be 1,400 pds. per sq. in. Applying our formula and remembering that $p = \frac{2\frac{1}{2}}{100}$ or $\frac{1}{40}$, we have $c' = 1400\ (1 + \frac{29}{40})$

$$= 2415 \text{ pds. per sq. in.}$$

With the stresses, that *did actually* occur at the elastic limit of the steel, as revealed by the graphs, this stress, 2415, may be compared. The table supplies the data:

Col.	Total Stress.	Stress in Steel.	Stress in Concrete.
1 B	1930	42,000	880
2 A	2400	42,000	1350
3 B	2680	42,000	1630
2 B	2055	42,000	1005
4 A	2370	42,000	1320
Average	2280	42,000	1230

We find therefore that the average total stress differs by less than 6 per cent. from the theoretic value 2415, and the average stress in the concrete, 1230, differs from its theoretic value 1400, by less than 12 per cent. Beyond this the formula should not be pressed. As to the distribution of the total stress between the steel and the concrete in the region beyond the elastic limit, nothing definite is known. Further, the value of n in such case is assuredly indeterminate and any assumption as to its value must in the nature of things be questionable. Considere succeeded in proving, with Coulomb's theorem as a basis, that the resistance given to sand by hooping is 2.4 times greater than the direct resistance of the longitudinal reinforcement of the same weight when the tensile stress in the former is equal to the compressive stress in the latter. He concluded that the same principle is true of concrete. The assumption was made, however, that the hoops are close together (from 1-4 to 1-10 of the diameter of the spiral). The efficiency of hooping depends on its capacity to resist lateral swelling and as the elastic lateral swelling is small, relative to the longitudinal strain, (Talbot gives Poisson's ratio for concrete as equal to 1-8) the hoops will certainly contribute little to the strength of a column for moderate loads. Such reinforcement is, however, quite effective in increasing the ultimate strength of a column, for, under increased loading, ultimate failure is prevented by the circumferential bands. Finally failure will occur by the excessive stretching of these bands or possibly by their rupture.

In the columns tested, series A were reinforced by hoops, series B by spirals having a pitch in each case of 3½ inches, equal, in fact, to the shorter diameter. Such a pitch is very much greater than Considere contemplated and hence it may be considered scarcely proper to institute a comparison between the values obtained from the equation and the strength as revealed in the testing machine. That the pitch was altogether too great was shown in at least one instance where, during the progress of the test, both concrete and the longitudinal rods bulged outward between two consecutive spirals. See photo, column 1-B.

In accordance with Considere's principle and considering also the presence of the longitudinal rods, the strength of the column will be given by the formula

$$P' = f\ (A_s + 2.4\ A_h) + \mathrm{A}\ (1 - p)\ c$$

where P' is the total load on column,
A_s is area of steel in longitudinals,
A_h is equivalent area of steel in hoops, if converted into longitudinals of same weight,
A is total area of cross-section and
c is compressive stress in concrete.

Now

$$P' = nc\ (A_s + 2.4\ A_h) + A\ (1 - p)\ c.$$

$$A_s = \frac{2\frac{1}{2}}{100} A \text{ and } 2.4 A_h = .012\ A.$$

When $c = 1400$, $n = 30$,

$$P' = 1400\ A\ [30\ (.025 + .012) + \frac{39}{40}]$$

or $P' = 1400\ A\ (1.11 + .975)$

$$= 1400\ A \times 2.085.$$

Dividing by A to obtain c', we have

$$c' = \frac{P'}{A} = 1400 \times 2.085$$

$$= 2926 \text{ pds. per sq. in.}$$

If for 1400, we substitute 1720,

$$c' = 3590 \text{ pds. per sq. in.}$$

The propriety of applying this equation in the second instance may be questioned on account of the uncertainty of the stresses produced in the longitudinal reinforcement, and in the hoops. Yet, even in the former case, the result, 2926 pds. per sq. in., is so much greater than the average stress actually reached, that it is doubtful if the hoops contributed anything to the strength of the columns. In other words, the presence of the longitudinals seems to be capable of accounting for the stresses actually obtained.

Two compressometer tests were made on plain concrete columns of the same sectional area, the span between collars being 15 inches. The stress-strain curves are shown in figures No. 1 and No. 2. For the purpose of comparison, a modulus of elasticity for each was determined for stresses of 500 pds. per sq. in. and also at a strain of .0014 as was done for the concrete of the reinforced columns. The results are given in the table following:

Plain Concrete.

No.	E_c at stress 500.	E_c at strain .0014	Ult. strength
1	2,630,000	1,060,000	1542
2	2,200,000	1,100,000	1744

Apparently the earlier modulus is somewhat larger than in the cases where the columns were reinforced. The later modulus, however, remains practically the same, showing that the hooping in this instance has contributed nothing to the stiffness or rigidity of the concrete which it encloses. We must not lose sight of the fact that the explanation of these phenomena is probably due to the wide spacing of the rings of metal. These tests as might be expected, lead to no conclusions as to the comparative merits of spirals over hoops or vice-versa.

Conclusions.

1. The employment of longitudinal steel reinforcement in concrete columns is not economical except where high working stresses in the concrete accompany a low value of E_c.

2. The hoops and spirals in the cases examined contributed little or nothing to the strength of the column beyond what could be explained by the presence of the longitudinal rods.

3. The hoops and spirals did not increase the rigidity of the concrete which they enclosed. These two phenomena may be explained by the fact that the pitch of the hoops was much greater than is considered good practice.

4. For columns up to 25 diameters, the tendency to fail by buckling under centrally applied loads is probably small.

5. For this particular class of concrete a modulus of elasticity of 1,500,000 for a stress of 500 pds. per sq. in. seems to be a fair value.

6. The variation in the ultimate strength values of the separate columns would indicate the necessity of employing a fairly high factor of safety in the use of reinforced concrete columns for construction purposes.

BABBITT BEARINGS FOR HIGH SPEEDS

HARRY H. ANGUS, B.A.Sc.

Until the advent of the steam turbine, high rotating speeds were seldom met with on large works, and when the turbine first came into use, the flexible shaft system of the De Laval turbine and the sleeve type of bearing used on the Parsons turbine were the only ones in common use. It was thought at that time that babbitt bearings could not be used successfully for high speeds on account of their inability to take up the vibrations caused by the rotating body being out of balance.

However, by using babbitt bearings for low speeds and gradually experimenting with them at higher speeds, it has been found quite practicable at the present time to use them with speeds of 2,500 R.P.M. or more. The taking up of the vibration is no doubt due to the fact that at high speed, the journal is separated from the bearing by a film of oil which thus allows it the same motion as in the sleeve type of bearing. Success in using babbitt bearing for high speed has no doubt also been due to some extent to the better facilities for balancing rotating bodies and also to the development of a special type of babbitt bearings suitable for this work.

In this class of work the clearance between the bearing and the shaft should range from .002 for 1″ shaft to .01 for 6″ shaft and .02 for shafts 12″ or more in diameter. If the clearance between the shaft and the bearing is too small the expansion of the shaft, if the speed is raised quickly, will cause binding. In the design of bearings it is usual to keep the peripheral velocity of the journal below 75 feet per second and the pressure below 80 lbs. per square inch of projected area of the bearing. Of course as the pressure is decreased, the speed may be increased or vice versa. In come recent tests with a large rotating body of 25 or 30 tons it was found that the bearing ran well with a pressure of 300 lbs. per square inch and a peripheral speed of 80 feet per second, which shows that the above limits are quite low. It is usual to make the length of the bearing three times the diameter as in ordinary bearings but probably 2½ to 1 or 2 to 1 would give better results.

Formerly a great many of the larger bearings were cored out for the purpose of supplying cooling water to the bearings. It has been found, however, that this is not necessary if the bearings are well proportioned and the oil supplied in generous quantities and well cooled before being used again. As bearings are usually enclosed in pedestals, not more than one-third of the heat generated is taken away by radiation and by air, leaving by far the greater part to be carried away by the oil. It is best in this work to have the oil flow into the bearing from a gravity system under a head of a few feet, thus insuring a very liberal supply of oil. After going through the bearing the oil is cooled and again pumped back into the gravity tank and being out of contact with the air is kept pure and can be used over again. In this way the same oil can be used for several months before an addition of new oil is necessary.

The bearing should be constructed on the ball and socket principle so as to readily take up the alignment and should also be provided with an accurate means of adjustment in any direction. If the bearings are not very carefully lined up there will be excessive vibration and heating. As there is no metallic contact in high speed bearings, the bearing metal is of small importance except that it is best to have some soft metal in order to save the shaft, in case of any failure in the supply of oil. It was found by test with a 15″x40″ bearing (see Transactions of American Society of Mechanical Engineers, Vol. 27) that the thickness of the oil film varied from .0019 at 460 R.P.M. to .0032 at 1050 R.P.M. and at about 300 R.P.M. there was metallic contact of the journal with the bearing which could be noticed owing to the rapid increase of friction around this point. These tests are cited merely to show that there is no metallic contact at high speed and that for this reason the quality of the bearing metal which is so important in slow speed bearings is of little importance with high speeds. It is, however, important that the babbitt metal be well fastened in the bearing and it is well to cast the babbitt in the bearing then rough bore and afterwards pean well and bore true to size.

In regard to the oil grooves, the best results will be obtained by having a single oil groove along the top of the bearing parallel to the axis of the shaft. The oil is fed to the centre of this groove and runs along it and is carried around to the surfaces in contact by centrifugal action. In some classes of bearings where there is no upward thrust it is possible to do away with the greater part of the babbitt in the top half of the bearing, thus allowing the oil to cool the shaft much more easily than otherwise.

Another point which it is well to look after is to see that the bearings are eased at their sides. This does away with any wedging action which may occur when the two halves of the bearing are bolted together and which is liable to cause binding, and in addition gives the oil a much better chance to do its work. The sharp edges of the bearings tend to wipe the oil from the shaft but if the bearing is eased away for about 25° above and below the centre line, the oil will enter the bearing surface nearly at a tangent and is thus more easily drawn in. It will be found that the loss in bearing surface area is more than balanced by these advantages.

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Editorial

A Reference Library

Improvement of technical courses is effected in many ways. Ability on the teaching staff, though a prime requisite, will not do all. There are certain aids which no efficient engineering college can afford to dispense with. For instance, in the teaching of Bridge and Structural Design, it has been found that collections of photographs, lantern slides and plans of typical structures are very helpful accessories. The larger Schools of Applied Science on this continent possess very fine equipment of this sort and not only are undergraduates and those academically connected benefited, but all interested persons have at their disposal, for purposes of reference, exceedingly valuable "libraries" of illustration. If such repositories had not been established by technical colleges, they probably would not yet exist, since much care and trouble must be taken in gathering together a large number of illustrations which will comprehensively represent the history and present practice of a given branch of engineering.

Recognizing the aid which would be afforded to those interested in Bridge and Structural Engineering by equipment of the above described kind, the Faculty of Applied Science and Engineering of the University of Toronto has recently brought its needs in this respect before a large number of bridge companies, and is pleased to find them willing to contribute valuable illustrations of their work as a nucleus to a collection. A number of interesting photographs have already arrived, and many more are likely to follow. It is hoped that those graduates who are engaged in bridge and structural work will supplement the collection by such photographs or plans as they may think of interest. The University is dependent largely upon the loyalty of its graduates for help of this kind, and more should be expected of them than of business corporations, many of which have no connection through their heads to our alma mater.

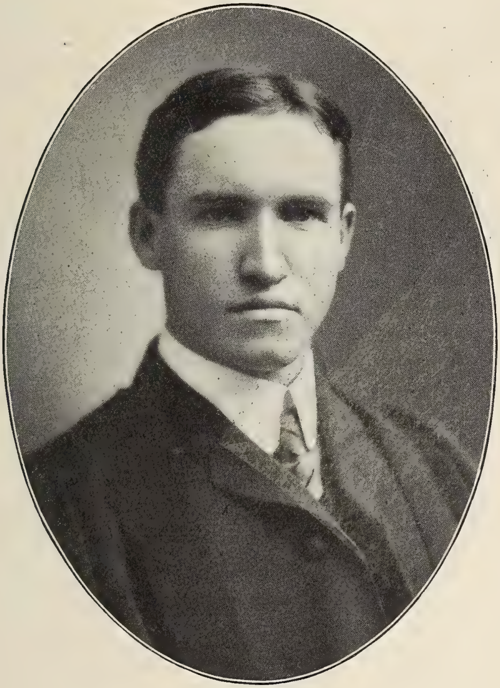
1908-09 Calendar.—The calendar of the Faculty of Applied Science for 1908-1909 has just been issued. Its contents have been entirely rearranged and improved. One item giving notice that after 1908 the course in Engineering may be lengthened by another year will interest all graduates. Copies may be had on application to the secretary.

OBITUARY

HARRY G. SMITH, B.A.Sc.

To many of those connected with the Faculty of Applied Science, whether as students or as members of the teaching staff, the early days of July brought a painful shock in the announcement that Harry G. Smith, B.A.Sc., '04, had lost his life in the waters of a lonely northern lake. To every one who had known him there came but one feeling — a comrade had gone down somewhere along the line.

After preparation at the St. Catharines Collegiate Institute, Mr. Smith entered the Faculty of Applied Science in the department of Mechanical and Electrical Engineering, graduating with the class of '03, and taking his degree in '04. Always a conscientious student, he frequently carried himself to the limits of endurance in his work. When under graduate days were over he became attached to the teaching staff as Fellow, and afterwards as Demonstrator, in Electrical Engineering, which position he held at the time of his death.



Harry G. Smith

This untimely close of a useful and promising life came while on a prospecting expedition to New Ontario, whither he had gone last May. After a long and arduous trip to Gray's Lake, he and his partner were returning by way of Obabika Lake, and were camped on the shores of Lake Wakimika when the unfortunate accident occurred. Mr. Smith was trolling alone in the canoe, when by some mischance the latter upset and threw him into the water, blowing out of his reach almost immediately. Entangled in the trolling line, as he was, the waters of Wakimika claimed him before aid arrived.

HARRY W. ARENS

Many friends in the University were shocked to hear of the death, during the summer, of a recent graduate, H. W. Arens. Mr. Arens received his early education in the public and high schools of Orillia. After leaving the high school he spent four years in the machine shop of the E. Long Co., Orillia, and came to the S. P. S. in the fall of 1901 to take the M. and E. course. He graduated in 1905, and on leaving the School went to Winnipeg. He was succeeding remarkably well in his profession when he fell a victim to consumption and had to give up all work last winter. His death took place at the home of his parents in June of last year.

OSBORNE CORRIGAN

Osborne Corrigan, of the third year School of Science, died at the Toronto General Hospital on May 6th of last year, of illness brought on while he was with a prospecting party in Northern Ontario. After completing the second year in the analytical chemistry course Mr. Corrigan left on a prospecting trip. When the party had reached a point some sixty miles north of Latchford, he was taken ill, and the only hope of saving his life was to get him to a hospital as quickly as possible. The task was undertaken by P. Lennox and Bert Rogers, assisted for part of the way by Herbert Aikens, of the third year in the School of Science. By five days of heavy sledging, impeded by great drifts of snow, the sick man was brought into touch with civilization, and soon after placed in the General Hospital. It was too late, however, and Mr. Corrigan soon after succumbed.

Corrigan stood well with his year, and his death came as a great personal loss to many men of the University.

Mr. Rogers and Mr. Lennox have since been honored by the Royal Humane Society for their brave attempt to save Mr. Corrigan's life.

OUR CONTRIBUTORS

C. H. C. WRIGHT, B.A.Sc.

Charles Henry Challenor Wright was born on board the British barque "Frank" in Chelsey Harbor, Mass., in 1864, his early childhood days being spent in Digby, N.S. His father, William Wright, was captain of a merchant liner, sailing from St. John and Boston, which was lost off the coast of the West Indies during the famous cyclone of 1867. There was but one other child, Minnie J. Wright, in the Wright family. C. H. C. had just got nicely started at the public school in Digby, N.S., when his mother married J. J. Cameron, a graduate of Dalhousie, and the family moved to Kingston, Ont., where Mr. Cameron proceeded to take a course in Theology at Queen's. During these three years C. H. C. attended the public school in Kingston until his stepfather accepted a charge in Shakespeare, Perth County, and Charles Henry had another taste of public school life, passing his entrance examination to the High School in 1878 and proceeded to attend the Collegiate at Vienna. In 1880 his stepfather left Shakespeare, removing to Pickering, Ont., and Wright entered the Pickering College, where he was soon to make the acquaintance of J. E. Bryant, its principal, from whom he received his mathematical inspirations. In 1881 he passed his Second Class Teachers' examinations. About this time he became inspired with the idea of coming to the "School" and during the fall term of 1881 paid special attention to Mathematics, Science, English, French, Latin and Art. In September, 1882, he went to the Model School in Whitby and in January, '83, began teaching in a school a mile and a half east of Pickering, where he remained until '85, living at home in Pickering and taking certain classes in Art at Pickering College under Professor Shrapnell. During this time our professor-to-be was keeping up his mathematics under the direction of some of the staff of Pickering College and incidentally playing football and cricket on every possible occasion, winning several of the field sports prizes, and filling in his spare time at the gymnasium, obtaining the championship in 1881 and 1882.

In the fall of 1885 he registered as a student at the School and his subsequent history is one of triumphs. There were thirty-five men entered the "Little Red Schoolhouse" which was very small in those days, the same year as C. H. C.

On the completion of his first year, out of twelve subjects, he captured ten first places; his second year exams. found him less fortunate, but still with nine places out of thirteen subjects, and in 1888 he graduated, together with sixteen others, obtaining twelve first places out of fifteen subjects and capturing the coveted prize in each year, which required not only that the student stand first in his class, but should also obtain an average of more than seventy-five per cent. of the possible marks.

During his second year he was appointed representative to the Engineering Society, and was made vice-president of the same society in his final year, Dr. Galbraith being president.

Wright was a member of the first tug-of-war team which won the trophy for the School, not to be surrendered again by them for many years. His vacations, while at the School, were spent with H. B. Proudfoot, Civil Engineer and Land Surveyor, and the summer of '88 with J. Tiernay, both in the Temiskaming district.

During the fall of '88 he taught in the Lindsay High School and in the spring of '89 went to Boston to work for Woodbury & Leighton, building contractors, as draughtsman, soon becoming chief of the estimating staff. In the fall of 1890 he secured the position of lecturer in Architecture at the School.

During his term as lecturer at the School he acted in conjunction with a committee from the Canadian Society of Civil Engineers, in an effort to establish a Canadian standard for cement tests and through experiments performed at the School was of great assistance in establishing a reputation for Canadian brands of Portland cement. He also carried on a series of tests on brick, brickwork, masonry, terracotta and stone, the results of his labors in stone tests being published by the Ontario Association of Architects, and those of the brickwork, masonry and terracotta published by the Engineering Society of the School. He also read a paper before the O. A. A. on "The Behavior of Steel under Stress," and later worked out an aspect compass for the architects. In Statics he discovered that the Vertical Shearing Force diagram represented the Bending Moment and developed a general solution for the Fink Roof Truss. Working in conjunction with W. J. Loudon he invented apparatus for the teaching of statics.

During the whole of his School days Prof. Wright was an all-round sport, playing Association football in various capacities on the foremost teams in the Dominion. It might be stated that even after he joined the staff of the School he continued to play football and even now his inclinations are in that direction. Those who were his fellow students say that he was a general favorite, active in everything in which the students were interested but seeming to have an insatiable desire for standing in front of the draughting board, sometimes being known to turn out nearly twice as many drawings as called for in the curriculum.

The Madawaska Club, organized in 1896, found an active member in Prof. Wright and if you wish to bring a smile to his lips ask him to tell you about the car-load of window-sash which he and W. J. Loudon took up Georgian Bay way and following on the heels of this will probably come some interesting information regarding the founding of the "Go Home" colony on the Go Home river.

Of the man as we know him now it may be said that every student has a friend in Prof. Wright and there is probably no member of the present staff who is more completely in touch with student affairs; he is energetic, powerful along lines needing executive ability, knows men well and how to handle them, kind-hearted and anxious to perform to the best of his ability anything which seems to be outlined to him as his duty. He has the interests of the School at heart and is giving his life to work out his ideals for the future of the Faculty of Applied Science. The fact that he is still a comparatively young man gives us cause for congratulation since we see for him many years to come of useful and efficient work.

B. E. FERNOW, DEAN OF THE FACULTY OF FORESTRY

Bernhard Eduard Fernow was born in the Province of Posen, Prussia, 1851. He graduated at the gymnasium at Bromberg; entered the profession of forestry, following the regular prescribed courses for Government service at Muenden Forest Academy, and also studying law at the University of Koenigsberg. He served his year in the army during the Franco-German War, 1870-71, and became lieutenant of reserves. In 1876 he emigrated to America, and engaged in metallurgical business, and later acted as consulting forest engineer. He served as secretary of the American Forestry Association (formerly Congress) 1883-87, and from 1887 as chairman of its Executive Committee, and later as its first vice-president. He was made Chief of the Division of Forestry of the United States Department of Agriculture at Washington in March, 1886, and continued in that office for twelve years, when in 1898 he became Director of the New York State College of Forestry at Cornell University, the first institution of its kind in the United States. After the close of this institution in 1903, due to political intrigue, he acted in private practice as Consulting Forest Engineer, besides giving several courses of lectures at the Yale Forest School. In 1907 he was called to Pennsylvania State College, to organize a forestry school on the same lines as the Cornell school, from which position he was called to Toronto University for the same purpose. He was made an honorary LL.D. by the State University of Wisconsin in 1897, and later by Queen's University. He was made a fellow of the American Association for the Advancement of Science; a life member of the American Institute of Mining Engineers; an honorary curator of the National Museum; an honorary member of the Scotch Arboricultural Association; of the Academy of Science of St. Petersburg; of the Pennsylvania Forestry Association, and an active member of various scientific societies of the United States. He is the author of Economics of Forestry, a full treatment of the subject, published by T. Y. Crowell & Co., New York; has completed a new book on History of Forests and Forestry in the United States and other countries, and is publishing the Forestry Quarterly, the only strictly professional journal of forestry in the United States. He has also been the author of many official reports, bulletins and other publications of professional or propagandist character, establishing the science of forestry in the United States.